Chapter 9
Combinations of Functions

Getting Started, p. 516

1. a) $f(-1) = (-1)^3 - 3(-1)^2 - 10(-1) + 24$
$= -1 - 3(1) + 10 + 24$
$= -1 - 3 + 10 + 24$
$= 30$

$f(4) = (4)^3 - 3(4)^2 - 10(4) + 24$
$= 64 - 3(16) - 40 + 24$
$= 64 - 48 - 40 + 24$
$= 0$

b) $f(-1) = \dfrac{4(-1)}{1-(-1)}$
$= \dfrac{-4}{1+1}$
$= \dfrac{-4}{2}$
$= -2$

$f(4) = \dfrac{4(4)}{1-(4)}$
$= \dfrac{16}{1-4}$
$= \dfrac{16}{-3}$
$= -5\frac{1}{3}$

c) $f(-1) = 3\log_{10}(-1)$
Since you cannot take the log of a negative number, the expression is undefined.
$f(4) = 3\log_{10}(4)$
$\doteq 3(0.6021)$
$\doteq 1.81$

d) $f(-1) = -5(0.5^{(-1-1)})$
$= -5(0.5^{-2})$
$= -5(4)$
$= -20$

$f(4) = -5(0.5^{(4-1)})$
$= -5(0.5^3)$
$= -5(0.125)$
$= -0.625$

2. The domain is the x-values. From the graph, the domain is $\{x \in \mathbf{R} \mid x \neq 1\}$.
The range is the y-values. From the graph, the range is $\{y \in \mathbf{R} \mid y \neq 2\}$.
There is no minimum or maximum value.
The function is never increasing.
The function is decreasing from $(-\infty, 1)$ and $(1, \infty)$.
The function approaches $-\infty$ as x approaches 1 from the left and ∞ as x approaches 1 from the right.
The vertical asymptote is $x = 1$.
The horizontal asymptote is $y = 2$.

3. a) The vertical stretch turns the function into $y = 2|x|$.
The translation 3 units to the right turns the function into $y = 2|x - 3|$.
b) The reflection in the x-axis turns the function into $y = -\cos(x)$.
The horizontal compression by a factor of $\frac{1}{2}$ turns the function into $y = -\cos(2x)$.
c) The reflection in the y-axis turns the function into $y = \log_3(-x)$.
The translation 4 units left makes the function $y = \log_3(-(x+4))$ or $\log_3(-x-4)$.
The translation 1 unit down turns the function into $y = \log_3(-x-4) - 1$.
d) The vertical stretch of 4 turns the function into $y = \frac{4}{x}$.
The reflection in the x-axis turns the function into $y = -\frac{4}{x}$.
The vertical translation 5 units down turns the function into $y = -\frac{4}{x} - 5$.

4. a) $2x^3 - 7x^2 - 5x + 4 = 0$

$$\begin{array}{r|rrrr} -1 & 2 & -7 & -5 & 4 \\ & \downarrow & -2 & 9 & -4 \\ \hline & 2 & -9 & 4 & 0 \end{array}$$

$2x^2 - 9x + 4 = 0$
$(2x - 1)(x - 4) = 0$
$2x - 1 = 0$ or $x - 4 = 0$
$2x = 1$ or $x = 4$
$x = \frac{1}{2}$ or $x = 4$

So, the solutions are $x = -1, \frac{1}{2}$, and 4

b)
$$\frac{2x+3}{x+3}+\frac{1}{2}=\frac{x+1}{x-1}$$
$$2(x+3)(x-1)\left(\frac{2x+3}{x+3}+\frac{1}{2}=\frac{x+1}{x-1}\right)$$
$$2(x-1)(2x+3)+(x+3)(x-1)=2(x+3)(x+1)$$
$$2(2x^2+x-3)+x^2+2x-3=2(x^2+4x+3)$$
$$4x^2+2x-6+x^2+2x-3=2x^2+8x+6$$
$$5x^2+4x-9=2x^2+8x+6$$
$$3x^2-4x-15=0$$
$$(3x+5)(x-3)=0$$
$$x=-\frac{5}{3}\text{ or }x=3$$

c) $\log x+\log(x-3)=1$
$$\log(x(x-3))=1$$
$$10^1=x(x-3)$$
$$10=x^2-3x$$
$$0=x^2-3x-10$$
$$0=(x-5)(x+2)$$
$$x=5\text{ or }x=-2$$
Cannot take the log of a negative number, so $x=5$.

d) $10^{-4x}-22=978$
$$10^{-4x}=1000$$
$$10^{-4x}=10^3$$
$$-4x=3$$
$$x=-\frac{3}{4}$$

e)
$$5^{x+3}-5^x=0.992$$
$$5^3(5^x)-5^x=0.992$$
$$5^x(5^3-1)=0.992$$
$$5^x(125-1)=0.992$$
$$5^x(124)=0.992$$
$$5^x=0.008$$
$$5^x=5^{-3}$$
$$x=-3$$

f)
$$2\cos^2 x=\sin x+1$$
$$2(\sin^2 x-1)=\sin x+1$$
$$2\sin^2 x-2=\sin x+1$$
$$2\sin^2 x-\sin x-3=0$$
$$(2\sin x-3)(\sin x+1)=0$$
$$2\sin x-3=0\text{ or }\sin x+1=0$$
$$\sin x=\frac{3}{2}\text{ or }\sin x=-1$$
Since $\sin x$ cannot be greater than 1, the first equation does not give a solution.
$\sin x=-1$
$x=270°$

5. a) $x^3-x^2-14x+24<0$
Find the critical points by solving
$x^3-x^2-14x+24=0.$
$x^3-x^2-14x+24=0$

2	1	−1	−14	24
	↓	2	2	−24
	1	1	−12	0

So, 2 is a critical value.
$$x^2+x-12=0$$
$$(x+4)(x-3)=0$$
$$x+4=0\text{ or }x-3=0$$
$$x=-4\text{ or }x=3$$
The critical values are -4, 2, and 3.
Test points that are in the intervals created by the critical values.
$(x-2)(x-3)(x+4)$
Test -5: $(-)(-)(-)=(-)<0$
Test 0: $(-)(-)(+)=(+)>0$
Test 2.5: $(+)(-)(+)=(-)<0$
Test 4: $(+)(+)(+)=(+)>0$
So, the solution is $(-\infty,-4)\cup(2,3)$

b) $\dfrac{(2x-3)(x-4)}{(x+2)}\geq 0$
Find the critical values.
$$2x-3=0$$
$$2x=3$$
$$x=\frac{3}{2}$$
$$x-4=0$$
$$x=4$$
$$x+2=0$$
$$x=-2$$
The critical values are $\frac{3}{2}$, 4, and -2
Test -3: $(-)(-)\div(-)=(-)<0$
Test 0: $(-)(-)\div(+)=(+)>0$

Test 2: $(+)(-) \div (+) = (-) < 0$
Test 5: $(+)(+) \div (+) = (+) > 0$
The solution is $(-2, \frac{3}{2}) \cup [4, \infty)$

6. a) $f(x) = 2 \sin (x - \pi)$
$f(-x) = 2 \sin (-x - \pi)$
$= -2 \sin (x - \pi)$
So, it is odd.

b) $f(x) = \dfrac{3}{4 - x}$
$f(-x) = \dfrac{3}{4 - (-x)}$
$= \dfrac{3}{4 + x}$
So, it is neither.

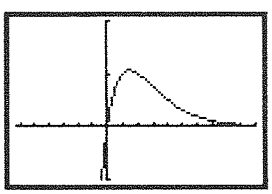
c) $f(x) = 4x^4 - 3x^2$
$f(-x) = 4(-x)^4 - 3(-x)^2$
$= 4x^4 - 3x^2$
So, it is even.

d) $f(x) = 2^{3x-1}$
$f(-x) = 2^{3(-x)-1}$
$= 2^{-3x-1}$
So, it is neither.

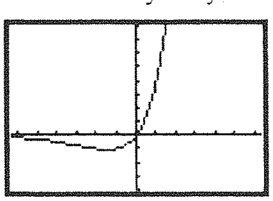
7. Polynomial, logarithmic, and exponential functions are continuous. Rational and trigonometric functions are sometimes continuous and sometimes not.

9.1 Exploring Combinations of Functions, p. 520

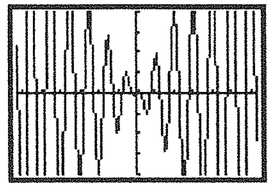
1. Answers may vary. For example, the graph of $y = ((\frac{1}{2})^x)(2x)$ is



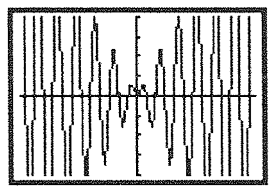
2. a) A function with a vertical asymptote and a horizontal asymptote:
If the functions $y = 2^x$ and $y = 2x$ are multiplied, the resulting function will have a vertical asymptote and a horizontal asymptote.
Answers may vary; for example, $y = (2^x)(2x)$;



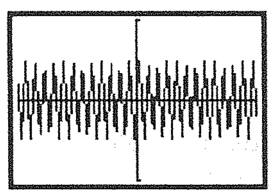
b) A function that is even:
$y = 2x$ is odd
$y = \cos (2\pi x)$ is odd
The product of the two functions will be even.
Answers may vary; for example,
$y = (2x)(\cos (2\pi x))$;



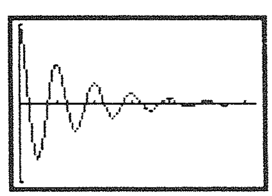
c) A function that is odd:
$y = 2x$ is odd
$y = \sin (2\pi x)$ is even
The product of the two functions is odd.
Answers may vary; for example,
$y = (2x)(\sin (2\pi x))$;



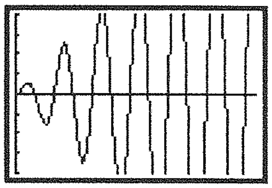
d) A function that is periodic:
$y = \sin 2\pi x$ is periodic
$y = \cos 2\pi x$ is periodic
The product of the two functions is periodic.
Answers may vary; for example,
$y = (\sin 2\pi x)(\cos 2\pi x)$;



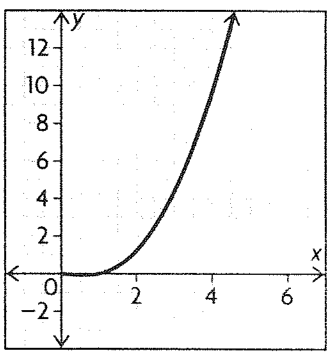
e) A function that resembles a periodic function with decreasing maximum values and increasing minimum values:
$y = \cos 2\pi x$ is periodic
$y = (\frac{1}{2})^x$ is decreasing
The product of the two functions will be a function that resembles a periodic function with decreasing maximum values and increasing minimum values
Answers may vary; for example, $y = (\frac{1}{2})^x(\cos 2\pi x)$ where $0 \le x \le 2\pi$;



f) A function that resembles a periodic function with increasing maximum values and decreasing minimum values:
$y = \sin 2\pi x$ is periodic
$y = 2x$ is increasing
The product of the two functions will be a function that resembles a periodic function with increasing maximum values and decreasing minimum values. Answers may vary; for example, $y = 2x \sin 2\pi x$ where $0 \le x \le 2\pi$;



3. Answers will vary. For example,
$y = x^2$
$y = \log x$
The product will be $y = x^2 \log x$.
Graph:



9.2 Combining Two Functions: Sums and Differences, pp. 528–530

1. a) $f + g = \{(-4, 4 + 2), (-2, 4 + 1), (1, 3 + 2), (4, 6 + 4)\}$
$= \{(-4, 6), (-2, 5), (1, 5), (4, 10)\}$

b) $g + f = \{(-4, 2 + 4), (-2, 1 + 4), (1, 2 + 3), (4, 4 + 6)\}$
$= \{(-4, 6), (-2, 5), (1, 5), (4, 10)\}$

c) $f - g = \{(-4, 4 - 2), (-2, 4 - 1), (1, 3 - 2), (4, 6 - 4)\}$
$= \{(-4, 2), (-2, 3), (1, 1), (4, 2)\}$

d) $g - f = \{(-4, 2 - 4), (-2, 1 - 3), (1, 2 - 3), (4, 4 - 6)\}$
$= \{(-4, -2), (-2, -3), (1, -1), (4, -2)\}$

e) $f + f = \{(-4, 4 + 4), (-2, 4 + 4), (1, 3 + 3), (3, 5 + 5), (4, 6 + 6)\}$
$= \{(-4, 8), (-2, 8), (1, 6), (3, 10), (4, 12)\}$

f) $g - g = \{(-4, 2 - 2), (-2, 1 - 1), (0, 2 - 2), (1, 2 - 2), (2, 2 - 2), (4, 4 - 4)\}$
$= \{(-4, 0), (-2, 0), (0, 0), (1, 0), (2, 0), (4, 0)\}$

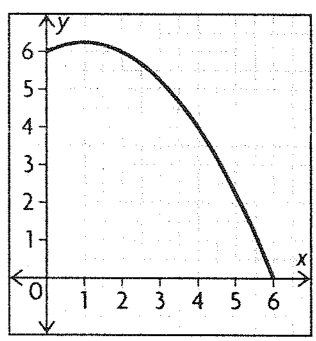
2. a)
$$f(4) = 4^2 - 3 = 16 - 3 = 13$$
$$g(4) = -\frac{6}{4 - 2} = -\frac{6}{2} = -3$$
So, $(f + g)(4) = 13 + -3 = 10$

b) 2; $(f + g)(x)$ is undefined at $x = 2$ because $g(x)$ is undefined at $x = 2$.

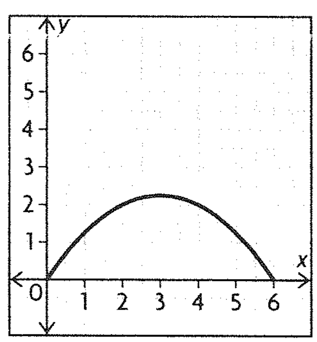
c) Since 2 cannot be part of the domain, the domain is $\{x \in \mathbf{R} | x \neq 2\}$

3. The domain of $f(x)$ is $x \ge -1$. The domain of $g(x)$ is $x < 1$. So the domain of $f - g$ is $\{x \in \mathbf{R} | -1 \le x < 1\}$.

4. To find the graph of $f + g$, add corresponding y-coordinates.
So, the graph should be:



To find the graph of $f - g$, subtract the corresponding y-coordinates.



5. a) $f + g = |x| + x$

b) $(f + g)(x) = |x| + x$

$(f + g)(-x) = |-x| + -x$

$= |x| - x$

The function is neither even or odd.

6. a) $f + g = \{(-6, 1 + 6), (-3, 7 + 3)\}$

$= \{(-6, 7), (-3, 10)\}$

b) $g + f = \{(-6, 6 + 1), (-3, 3 + 7)\}$

$= \{(-6, 7), (-3, 10)\}$

c) $f - g = \{(-6, 1 - 6), (-3, 7 - 3)\}$

$= \{(-6, -5), (-3, 4)\}$

d) $g - f = \{(-6, 6 - 1), (-3, 3 - 7)\}$

$= \{(-6, 5), (-3, -4)\}$

e) $f - f = \{(-9, -2 - (-2)), (-8, 5 - 5), (-6, 1 - 1), (-3, 7 - 7), (-1, -2 - (-2)), (0, -10 - (-10))\}$

$= \{(-9, 0), (-8, 0), (-6, 0), (-3, 0), (-1, 0), (0, 0)\}$

f) $g + g = \{(-7, 7 + 7), (-6, 6 + 6), (-5, 5 + 5), (-4, 4 + 4), (-3, 3 + 3)\}$

$= \{(-7, 14), (-6, 12), (-5, 10), (-4, 8), (-3, 6)\}$

7. a) $(f + g)(x) = \dfrac{1}{3x + 4} + \dfrac{1}{x - 2}$

$= \dfrac{x - 2}{(3x + 4)(x - 2)} + \dfrac{3x + 4}{(3x + 4)(x - 2)}$

$= \dfrac{4x + 2}{(3x + 4)(x - 2)}$

$= \dfrac{2(2x + 1)}{3x^2 - 2x - 8}$

b) The denominator cannot be 0, so $3x + 4 \neq 0$ or $x \neq -\frac{4}{3}$ and $x - 2 \neq 0$ or $x \neq 2$.

So, the domain is $\{x \in \mathbf{R} \mid x \neq -\frac{4}{3} \text{ or } 2\}$

c) $(f + g)(8) = \dfrac{2(2(8) + 1)}{3(8)^2 - 2(8) - 8}$

$= \dfrac{2(17)}{3(64) - 16 - 8}$

$= \dfrac{34}{192 - 16 - 8}$

$= \dfrac{34}{168}$

$= \dfrac{17}{84}$

d) $(f - g)(8) = \dfrac{1}{3(8) + 4} - \dfrac{1}{(8) - 2}$

$= \dfrac{1}{24 + 4} - \dfrac{1}{6}$

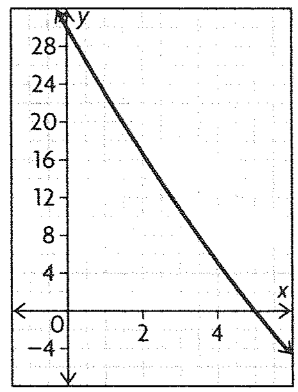
$= \dfrac{1}{28} - \dfrac{1}{6}$

$= \dfrac{3}{84} - \dfrac{14}{84}$

$= -\dfrac{11}{84}$

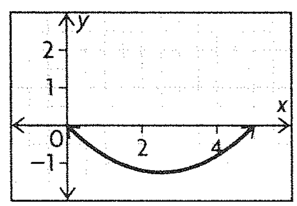
8. To find the graph of $f + g$, add corresponding y-coordinates.

So, the graph should be:

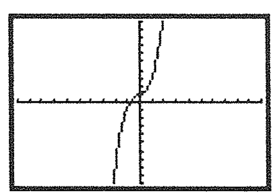


To find the graph of $f - g$, subtract corresponding y-coordinates.

So, the graph should be:



9. a) $f(x) + g(x) = 2^x + x^3$



symmetry: The function is not symmetric.

increasing/decreasing: The function is always increasing.

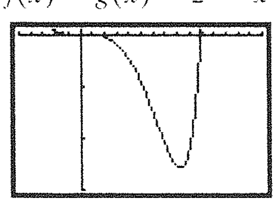
zeros: $x = -0.8262$

maximum/minimum: no maximum or minimum

period: N/A

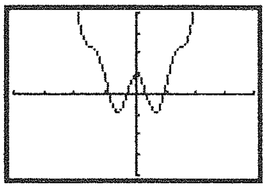
domain and range: The domain is all real numbers. The range is all real numbers.

$f(x) - g(x) = 2^x - x^3$



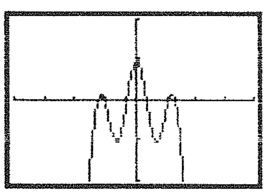
symmetry: The function is not symmetric.
increasing/decreasing: The function is always decreasing.
zeros: $x = 1.3735$
maximum/minimum: no maximum or minimum
period: N/A
domain and range: The domain is all real numbers. The range is all real numbers.

b) $f(x) + g(x) = \cos(2\pi x) + x^4$



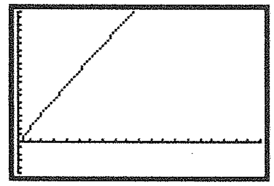
symmetry: The function is symmetric across the line $x = 0$.
increasing/decreasing: The function is decreasing from $-\infty$ to -0.4882 and 0 to 0.4882 and increasing from -0.4882 to 0 and 0.4882 to ∞.
zeros: $x = -0.7092, -0.2506, 0.2506, 0.7092$
maximum/minimum: relative maximum at $x = 0$ and relative minimums at $x = -0.4882$ and $x = 0.4882$
period: N/A
domain and range: The domain is all real numbers. The range is all real numbers greater than -0.1308.

$f(x) - g(x) = \cos(2\pi x) - x^4$



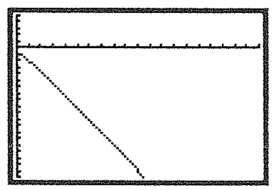
symmetry: The function is symmetric across the line $x = 0$.
increasing/decreasing: The function is increasing from $-\infty$ to -0.9180 and -0.5138 to 0 and 0.5138 to 0.9180; decreasing from -0.9180 to -0.5138 and 0 to 0.5138 and 0.9180 to ∞.
zeros: $x = -1, -0.8278, -0.2494, 0.2494, 0.8278, 1$
maximum/minimum: relative maxima at -0.9180, 0, and 0.9180; relative minima at -0.5138 and 0.5138
period: N/A
domain and range: The domain is all real numbers. The range is all real numbers less than 1.

c) $f(x) + g(x) = \log(x) + 2x$



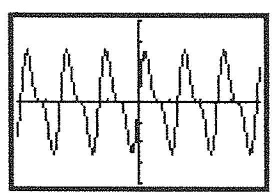
symmetry: The function is not symmetric.
increasing/decreasing: The function is increasing from 0 to ∞.
zeros: none
maximum/minimum: none
period: N/A
domain and range: The domain is all real numbers greater than 0. The range is all real numbers.

$f(x) - g(x) = \log(x) - 2x$



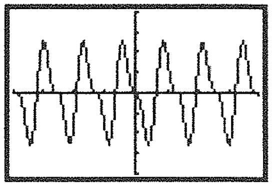
symmetry: The function is not symmetric.
increasing/decreasing: The function is increasing from 0 to approximately 0.2 and decreasing from approximately 0.2 to ∞.
zeros: none
maximum/minimum: maximum at $x \doteq 0.2$
period: N/A
domain and range: The domain is all real numbers greater than 0. The range is all real numbers less than or equal to approximately -1.1.

d) $f(x) + g(x) = \sin(2\pi x) + 2\sin(\pi x)$



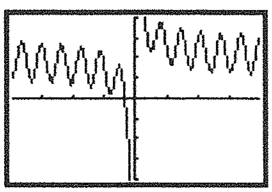
symmetry: The function is symmetric about the origin.
increasing/decreasing: The function is increasing from $-0.33 + 2k$ to $0.33 + 2k$ and decreasing from $0.33 + 2k$ to $1.67 + 2k$.
zeros: k
maximum/minimum: minimum at $x = -0.33 + 2k$ and maximum at $x = 0.33 + 2k$

period: 2
domain and range: The domain is all real numbers. The range is all real numbers between -2.598 and 2.598.
$f(x) - g(x) = \sin(2\pi x) - 2\sin(\pi x)$



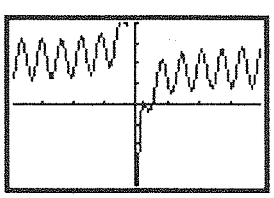
symmetry: The function is symmetric about the origin.
increasing/decreasing: $0.67 + 2k$ to $1.33 + 2k$ and decreasing from $-0.67 + 2k$ to $0.67 + 2k$
zeros: k
maximum/minimum: minimum at $0.67 + 2k$ and maximum at $1.33 + 2k$
period: 2
domain and range: The domain is all real numbers. The range is all real numbers between -2.598 to 2.598.

e) $f(x) + g(x) = \sin(2\pi x) + \dfrac{1}{x}$



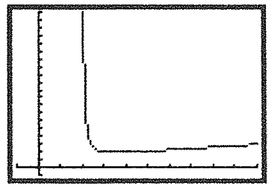
symmetry: The function is not symmetric.
increasing/decreasing: The function is increasing and decreasing at irregular intervals.
zeros: the zeros are changing at irregular intervals.
maximum/minimum: the maximums and minimums are changing at irregular intervals
period: N/A
domain and range: The domain is all real numbers except 0. The range is all real numbers.

$f(x) - g(x) = \sin(2\pi x) - \dfrac{1}{x}$



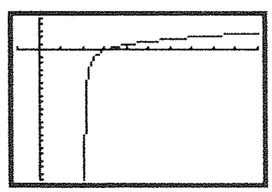
symmetry: The function is not symmetric.
increasing/decreasing: The function is increasing and decreasing at irregular intervals.
zeros: The zeros are changing at irregular intervals.
maximum/minimum: The maximums and minimums are changing at irregular intervals.
period: N/A
domain and range: The domain is all real numbers except 0. The range is all real numbers.

f) $f(x) + g(x) = \sqrt{x-2} + \dfrac{1}{x-2}$



symmetry: The function is not symmetric.
increasing/decreasing: The function is increasing from 3.5874 to ∞ and decreasing from 2 to 3.5874.
zeros: none
maximum/minimum: minimum at $x = 3.5874$
period: N/A
domain and range: The domain is all real numbers greater than 2. The range is all real numbers greater than 1.8899.

$f(x) = g(x) = \sqrt{x-2} - \dfrac{1}{x-2}$



symmetry: The function is not symmetric.
increasing/decreasing: The function is increasing from 2 to ∞.
zeros: $x = 3$
maximum/minimum: none
period: N/A
domain and range: The domain is all real numbers greater than 2. The range is all real numbers.

10. a) The sum of two even functions will be even because replacing x with $-x$ will still result in the original function.
b) The sum of two odd functions will be odd because replacing x with $-x$ will still result in the opposite of the original function.
c) The sum of an even and an odd function will result in neither an even or an odd function because replacing x with $-x$ will not result in the same function or in the opposite of the function.

11. a) $R(t) = 5000 - 25t - 1000\cos\left(\frac{\pi}{6}t\right)$;
it is neither odd nor even; it is increasing during the first 6 months of each year and decreasing during the last 6 months of each year; it has one zero, which is the point at which the deer population has become extinct; it has a maximum value of 3850 and a minimum value of 0, so its range is $\{R(t) \in \mathbf{R} \mid 0 \le R(t) \le 3850\}$.

b) $R(t) = 5000 - 25t - 1000\cos\left(\frac{\pi}{6}t\right)$

$$0 = 5000 - 25t - 1000\cos\left(\frac{\pi}{6}t\right)$$

Use a graphing calculator to find the zero of the function.
The deer population is extinct after about 167 months, or 13 years and 11 months.

12. The stopping distance can be defined by the function $s(x) = 0.006x^2 + 0.21x$.
If the vehicle is travelling at 90 km/h, the stopping distance is:

$$\begin{aligned} s(90) &= 0.006(90)^2 + 0.21(90) \\ &= 0.006(8100) + 18.9 \\ &= 48.6 + 18.9 \\ &= 67.5 \text{ m} \end{aligned}$$

13. $f(x) = \sin(\pi x)$; $g(x) = \cos(\pi x)$

14. The function is neither even nor odd; it is not symmetrical with respect to the y-axis or with respect to the origin; it extends from the third quadrant to the first quadrant; it has a turning point between $-n$ and 0 and another turning point at 0; it has zeros at $-n$ and 0; it has no maximum or minimum values; it is increasing when $x \in (-\infty, -n)$ and when $x \in (0, \infty)$; when $x \in (-n, 0)$, it increases, has a turning point, and then decreases; its domain is $\{x \in \mathbf{R}\}$, and its range is $\{y \in \mathbf{R}\}$.

15. a) $f(x) = 0$; $g(x) = 0$

b) $f(x) = x^2$; $g(x) = x^2$

$$\begin{aligned} (f + g)(x) &= x^2 + x^2 \\ &= 2x^2 \end{aligned}$$

It is a vertical stretch of 2 from the original function.

c) $f(x) = \frac{1}{x - 2}$; $g(x) = \frac{1}{x - 2} + 2$

$$\begin{aligned} \text{So, } (f - g)(x) &= \frac{1}{x - 2} - \left(\frac{1}{x - 2} + 2\right) \\ &= -2 \end{aligned}$$

16.
$$\begin{aligned} h(x) &= x^2 - nx + 5 + mx^2 + x - 3 \\ &= (1 + m)x^2 + (1 - n)x + 2 \end{aligned}$$

$$\begin{aligned} 3 &= (1 + m)(1)^2 + (1 - n)(1) + 2 \\ 3 &= 1 + m + 1 - n + 2 \\ 3 &= 4 + m - n \\ -1 &= m - n \text{ EQUATION 1} \\ 18 &= (1 + m)(-2)^2 + (1 - n)(-2) + 2 \\ 18 &= (1 + m)(4) - 2 + 2n + 2 \\ 18 &= 4 + 4m + 2n \\ 14 &= 4m + 2n \text{ EQUATION 2} \\ -1 &= m - n \\ 14 &= 4m + 2n \\ 2(-1 &= m - n) \\ 14 &= 4m + 2n \\ -2 &= 2m - 2n \\ 14 &= 4m + 2n \\ 12 &= 6m \\ 2 &= m \\ -1 &= 2 - n \\ -3 &= -n \\ 3 &= n \end{aligned}$$

9.3 Combining Two Functions: Products, pp. 537–539

1. a)
$$\begin{aligned} (f \times g)(x) &= \{(0, 2 \times -1), (1, 5 \times -2), (2, 7 \times 3), (3, 12 \times 5)\} \\ &= \{(0, -2), (1, -10), (2, 21), (3, 60)\} \end{aligned}$$

b)
$$\begin{aligned} (f \times g)(x) &= \{(0, 3 \times 4), (2, 10 \times -2)\} \\ &= \{(0, 12), (2, -20)\} \end{aligned}$$

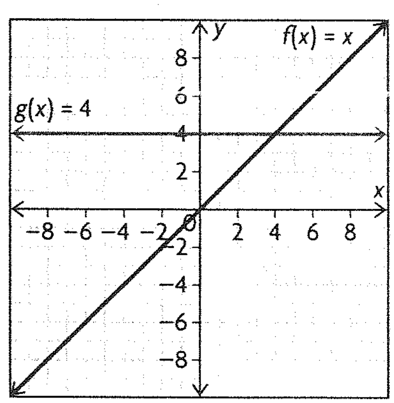
c)
$$\begin{aligned} (f \times g)(x) &= x(4) \\ &= 4x \end{aligned}$$

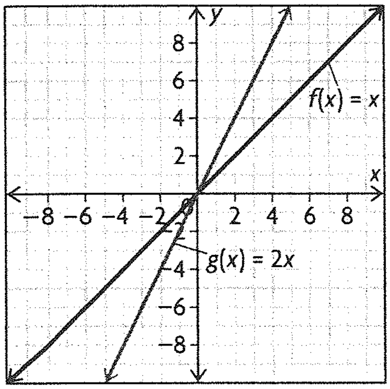
d)
$$\begin{aligned} (f \times g)(x) &= x(2x) \\ &= 2x^2 \end{aligned}$$

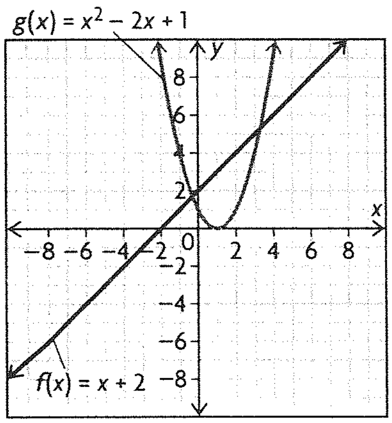
e)
$$\begin{aligned} (f \times g)(x) &= (x + 2)(x^2 - 2x + 1) \\ &= x^3 - 2x^2 + x + 2x^2 - 4x + 2 \\ &= x^3 - 3x + 2 \end{aligned}$$

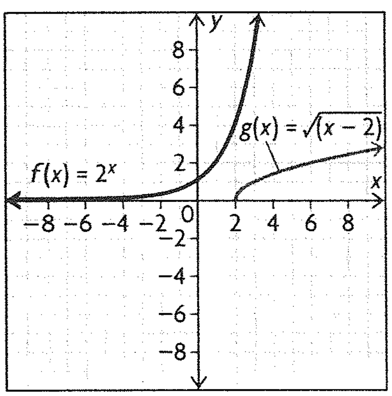
f)
$$\begin{aligned} (f \times g)(x) &= 2^x\left(\sqrt{x - 2}\right) \\ &= 2^x\sqrt{x - 2} \end{aligned}$$

2. a) 1(c):



1(d): 

1(e): 

1(f): 

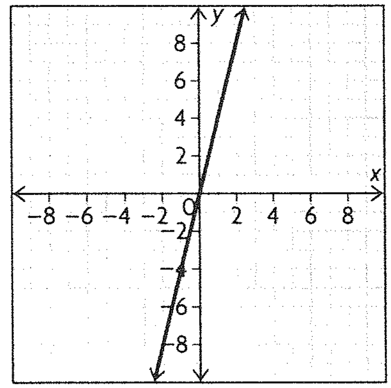
b) 1(c): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R}\}$

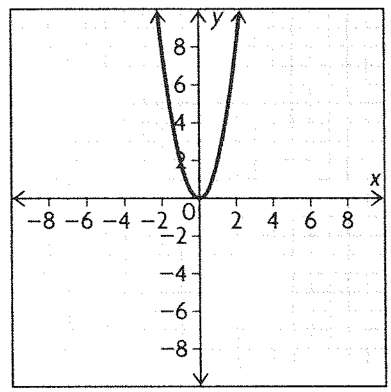
1(d): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R}\}$

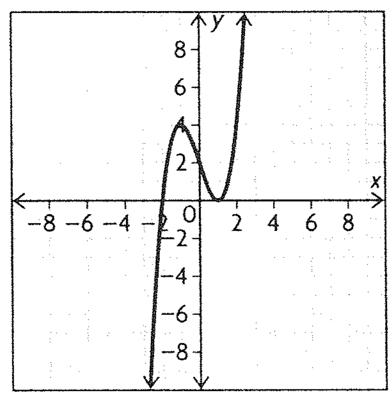
1(e): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R}\}$

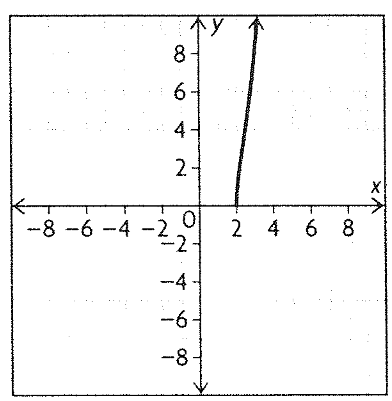
1(f): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R} \mid x \geq 2\}$

c) The graph of $f \times g$ can be found by multiplying corresponding y-coordinates.

1(c): 

1(d): 

1(e): 

1(f): 

d) 1(c): $\{x \in \mathbf{R}\}$
1(d): $\{x \in \mathbf{R}\}$
1(e): $\{x \in \mathbf{R}\}$
1(f): $\{x \in \mathbf{R} | x \geq 2\}$

3. $(f \times g)(x) = (\sqrt{1+x})(\sqrt{1-x})$
So, $x \geq -1$ and $x \leq 1$ since the radicand must be greater than or equal to 0.
So, the domain is $\{x \in \mathbf{R} | -1 \leq x \leq 1\}$

4. a) $(f \times g)(x) = (x-7)(x+7)$
$= x^2 - 49$

b) $(f \times g)(x) = (\sqrt{x+10})(\sqrt{x+10})$
$= x + 10$

c) $(f \times g)(x) = 7x^2(x-9)$
$= 7x^3 - 63x^2$

d) $(f \times g)(x) = (-4x-7)(4x+7)$
$= -16x^2 - 28x - 28x - 49$
$= -16x^2 - 56x - 49$

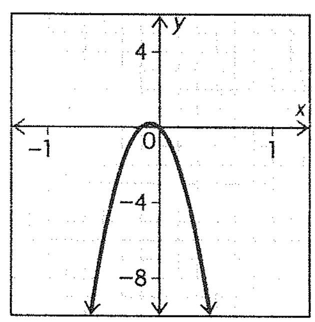
e) $(f \times g)(x) = 2\sin x\left(\frac{1}{x-1}\right)$
$= \frac{2\sin x}{x-1}$

f) $(f \times g)(x) = \log(x+4)(2^x)$
$= 2^x \log(x+4)$

5. 4(a): D $= \{x \in \mathbf{R}\}$; R $= \{y \in \mathbf{R} | y \geq -49\}$
4(b): D $= \{x \in \mathbf{R} | x \geq -10\}$;
R $= \{y \in \mathbf{R} | y \geq 0\}$
4(c): D $= \{x \in \mathbf{R}\}$; R $= \{y \in \mathbf{R}\}$
4(d): D $= \{x \in \mathbf{R}\}$; R $= \{y \in \mathbf{R} | y \leq 0\}$
4(e): D $= \{x \in \mathbf{R} | x \neq -1\}$; R $= \{y \in \mathbf{R}\}$
4(f): D $= \{x \in \mathbf{R} | x > -4\}$; R $= \{y \in \mathbf{R} | y \geq 0\}$

6. 4(a): symmetry: The function is symmetric about the line $x = 0$.
increasing/decreasing: The function is increasing from 0 to ∞. The function is decreasing from $-\infty$ to 0.
zeros: $x = -7, 7$
maximum/minimum: The minimum is at $x = 0$.
period: N/A
4(b): symmetry: The function is not symmetric.
increasing/decreasing: The function is increasing from -10 to ∞.
zeros: $x = -10$
maximum/minimum: The minimum is at $x = -10$.
period: N/A
4(c): symmetry: The function is not symmetric.
increasing/decreasing: The function is increasing from $-\infty$ to 0 and from 6 to ∞.
zeros: $x = 0, 9$
maximum/minimum: The relative minimum is at $x = -6$. The relative maximum is at $x = 0$.
period: N/A
4(d): symmetry: The function is symmetric about the line $x = -1.75$.
increasing/decreasing: The function is increasing from $-\infty$ to -1.75 and is decreasing from -1.75 to ∞.
zero: $x = -1.75$
maximum/minimum: The maximum is at $x = -1.75$.
period: N/A
4(e): symmetry: The function is not symmetric.
increasing/decreasing: The function is increasing from $-\infty$ to 0 and from 6 to ∞.
zeros: $x = 0, 9$
maximum/minimum: The relative minima are at $x = -4.5336$ and 4.4286. The relative maximum is at $x = -1.1323$.
period: N/A
4(f): symmetry: The function is not symmetric.
increasing/decreasing: The function is increasing from -4 to ∞.
zeros: none
maximum/minimum: none
period: N/A

7. $f(x) = -4x$
$g(x) = 6x + 1$
$(f \times g)(x) = -4x(6x+1)$
$= -24x^2 - 4x$



8. a) $\left\{x \in \mathbf{R} \middle| x \neq -2, 7, \frac{\pi}{2}, \text{ or } \frac{3\pi}{2}\right\}$
b) $\{x \in \mathbf{R} | x > 8\}$
c) $\{x \in \mathbf{R} | x \geq -81 \text{ and } x \neq 0, \pi, \text{ or } 2\pi\}$
d) $\{x \in \mathbf{R} | x \leq -1 \text{ or } x \geq 1 \text{ and } x \neq -3\}$

9. $(f \times p)(t)$ represents the total energy consumption in a particular country at time t

10. a) $R(x) = (20\,000 - 750x)(25 + x)$ or $R(x) = 500\,000 + 1250x - 750x^2$, where x is the increase in the admission fee in dollars
b) Yes, it's the product of the function $P(x) = 20\,000 - 750x$, which represents the number of daily visitors, and $F(x) = 25 + x$, which represents the admission fee.

c) Use a graphing calculator. The ticket price that will maximize revenue is \$25.83.

11. $m(t) = ((0.9)^t)(650 + 300t)$

Use a graphing calculator to estimate.

The amount of contaminated material is at its greatest after about 7.3 s.

12. The statement is false. If $f(x)$ and $g(x)$ are odd functions, then their product will always be an even function. The reason is because when you multiply a function that has an odd degree with another function that has an odd degree, you add the exponents, and when you add two odd numbers together, you get an even number.

13.

$$\begin{aligned}
h(x) &= (mx^2 + 2x + 5)(2x^2 - nx - 2)\\
-40 &= (m(1)^2 + 2(1) + 5)\\
&\quad \times (2(1)^2 - n(1) - 2)\\
-40 &= (m + 2 + 5)(2 - n - 2)\\
-40 &= (m + 7)(-n)\\
\frac{40}{m + 7} &= n \text{ EQUATION 1}\\
24 &= (m(-1)^2 + 2(-1) + 5)\\
&\quad \times (2(-1)^2 - n(-1) - 2)\\
24 &= (m - 2 + 5)(2 + n - 2)\\
24 &= (m + 3)(n)\\
\frac{24}{m + 3} &= n \text{ EQUATION 2}\\
\frac{40}{m + 7} &= \frac{24}{m + 3}\\
40(m + 3) &= 24(m + 7)\\
40m + 120 &= 24m + 168\\
16m &= 48\\
m &= 3\\
\frac{24}{3 + 3} &= n\\
\frac{24}{6} &= n\\
4 &= n
\end{aligned}$$

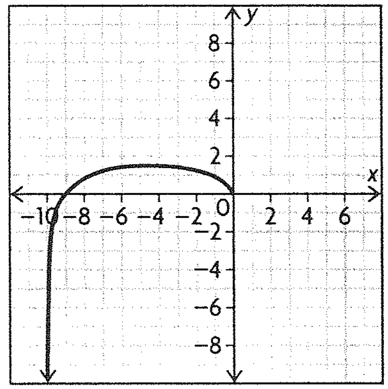
So, the equations are $f(x) = 3x^2 + 2x + 5$ and $g(x) = 2x^2 - 4x - 2$.

14. a) $(f \times g)(x) = \sqrt{-x}\log(x + 10)$

The domain is $\{x \in \mathbf{R} \mid -10 < x \leq 0\}$.

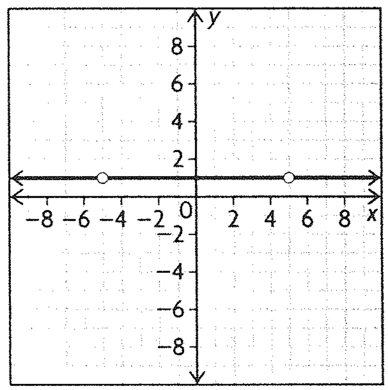
b) One strategy is to create a table of values for $f(x)$ and $g(x)$ and to multiply the corresponding y-values together. The resulting values could then be graphed. Another strategy is to graph $f(x)$ and $g(x)$ and to then create a graph for $(f \times g)(x)$ based on these two graphs. The first strategy is probably better than the second strategy, since the y-values for $f(x)$ and $g(x)$ will not be round numbers and will not be easily discernable from the graphs of $f(x)$ and $g(x)$.

c)



15. a) $f(x) \times \dfrac{1}{f(x)} = (x^2 - 25) \times \dfrac{1}{x^2 - 25}$
$= 1$

b) The domain of the function is $\{x \in \mathbf{R} \mid x \neq -5 \text{ or } 5\}$.



c) The range will always be 1. If f is of odd degree, there will always be at least one value that makes the product undefined and which is excluded from the domain. If f is of even degree, there may be no values that are exlcuded from the domain.

16. a)

$$\begin{aligned}
f(x) &= 2^x\\
g(x) &= x^2 + 1\\
(f \times g)(x) &= 2^x(x^2 + 1)
\end{aligned}$$

b)

$$\begin{aligned}
f(x) &= x\\
g(x) &= \sin(2\pi x)\\
(f \times g)(x) &= x\sin(2\pi x)
\end{aligned}$$

17. a)

$$\begin{aligned}
4x^2 - 91 &= (2x + 9)(2x - 9)\\
f(x) &= (2x + 9)\\
g(x) &= (2x - 9)
\end{aligned}$$

b)

$$\begin{aligned}
8\sin^3 x + 27 &= (2\sin x + 3)\\
&\quad \times (4\sin^2 x - 6\sin x + 9)\\
f(x) &= (2\sin x + 3)\\
g(x) &= (4\sin^2 x - 6\sin x + 9)
\end{aligned}$$

c)

$$\begin{aligned}
4x^{\frac{11}{2}} - 3x^{\frac{7}{2}} + x^{\frac{1}{2}} &= x^{\frac{1}{2}}(4x^5 - 3x^3 + 1)\\
f(x) &= x^{\frac{1}{2}}\\
g(x) &= (4x^5 - 3x^3 + 1)
\end{aligned}$$

d) $\dfrac{6x - 5}{2x + 1} = \dfrac{1}{2x + 1} \times 6x - 5$

$f(x) = \dfrac{1}{2x + 1}$

$g(x) = 6x - 5$

9.4 Exploring Quotients of Functions, p. 542

1. a) $(f \div g)(x) = \dfrac{5}{x}, x \neq 0$

b) $(f \div g)(x) = \dfrac{4x}{2x - 1}, x \neq \dfrac{1}{2}$

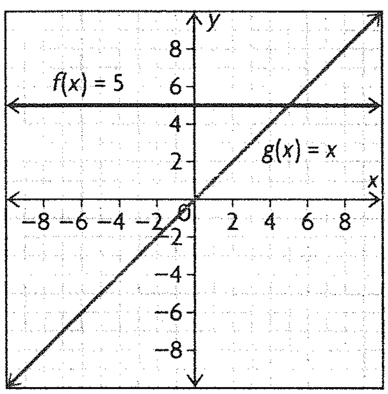
c) $(f \div g)(x) = \dfrac{4x}{x^2 + 4}$

d) $(f \div g)(x) = \dfrac{(x + 2)(\sqrt{x - 2})}{x - 2}, x > 2$

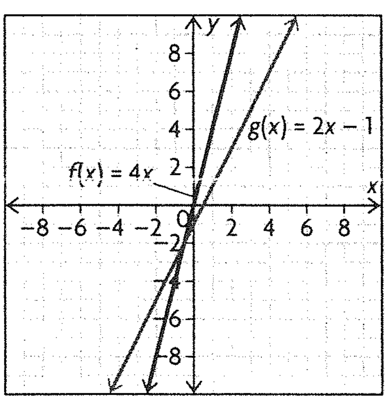
e) $(f \div g)(x) = \dfrac{8}{1 + \left(\frac{1}{2}\right)^x}$

f) $(f \div g)(x) = \dfrac{x^2}{\log(x)}, x > 0$

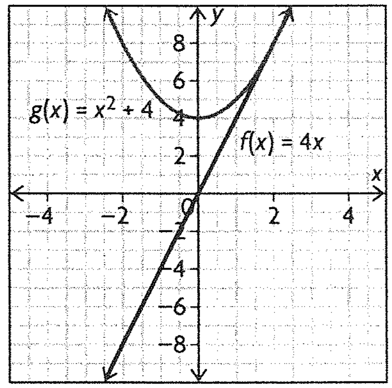
2. a) 1(a):



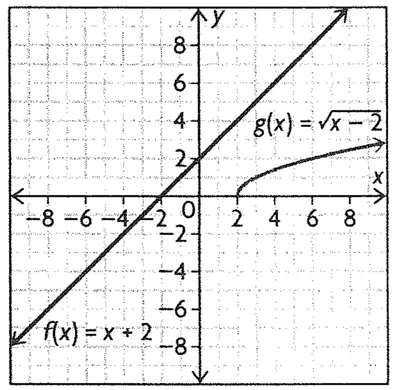
1(b):



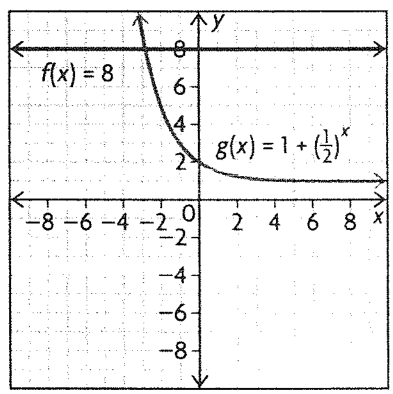
1(c):



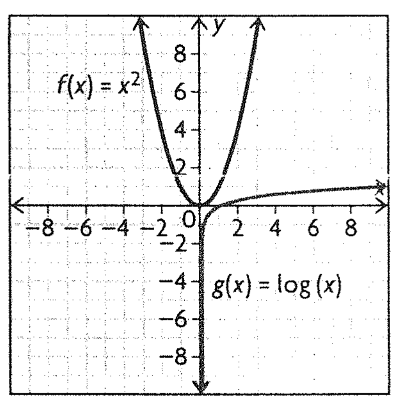
1(d):



1(e):

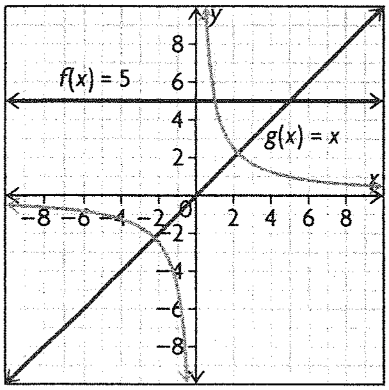


1(f):



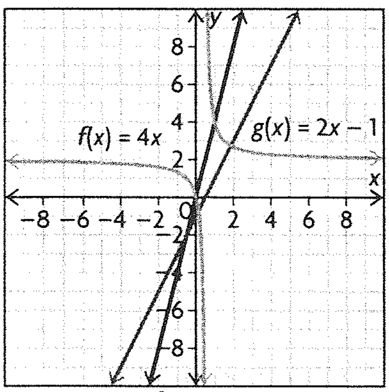
b) 1(a): domain of f: $\{x \in \mathbf{R}\}$; domain of g: $\{x \in \mathbf{R}\}$
1(b): domain of f: $\{x \in \mathbf{R}\}$; domain of g: $\{x \in \mathbf{R}\}$
1(c): domain of f: $\{x \in \mathbf{R}\}$; domain of g: $\{x \in \mathbf{R}\}$
1(d): domain of f: $\{x \in \mathbf{R}\}$; domain of g: $\{x \in \mathbf{R} \mid x \geq 2\}$
1(e): domain of f: $\{x \in \mathbf{R}\}$; domain of g: $\{x \in \mathbf{R}\}$
1(f): domain of f: $\{x \in \mathbf{R}\}$; domain of g: $\{x \in \mathbf{R} \mid x > 0\}$

c) 1(a):



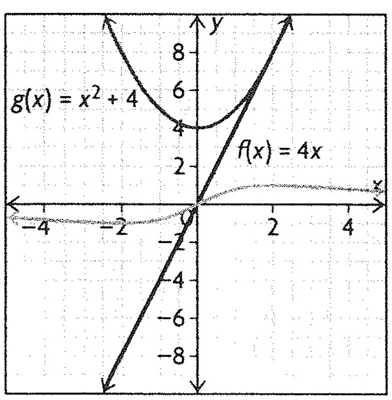
$y = \left(\frac{f}{g}\right)(x) = \frac{5}{x}$

1(b):



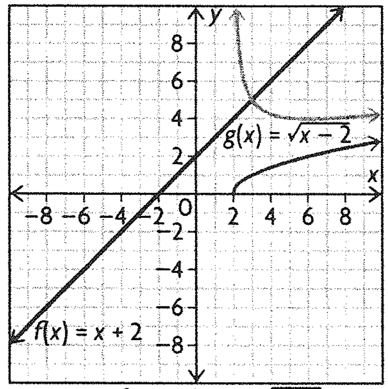
$y = \left(\frac{f}{g}\right)(x) = \frac{4x}{2x - 1}$

1(c):



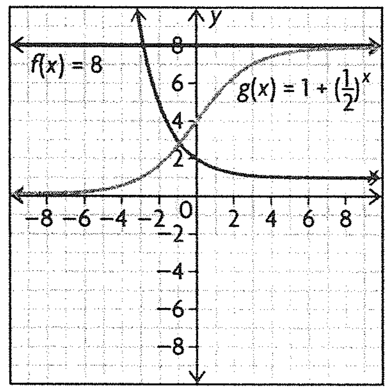
$y = \left(\frac{f}{g}\right)(x) = \frac{4x}{x^2 + 4}$

1(d):



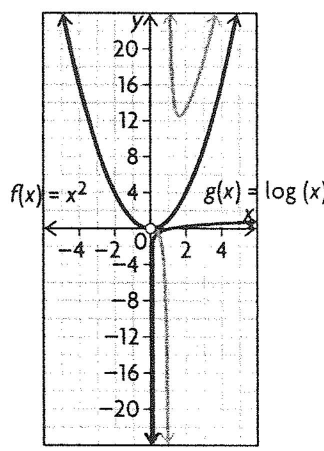
$y = \left(\frac{f}{g}\right)(x) = \frac{(x + 2)\sqrt{x - 2}}{x - 2}$

1(e):



$y = \left(\frac{f}{g}\right)(x) = \frac{8}{1 + \left(\frac{1}{2}\right)^x}$

1(f):



$y = \left(\frac{f}{g}\right)(x) = \frac{x^2}{\log(x)}$

d) 1(a): domain of $(f \div g)$: $\{x \in \mathbf{R} \mid x \neq 0\}$
1(b): domain of $(f \div g)$: $\left\{x \in \mathbf{R} \mid x \neq \frac{1}{2}\right\}$
1(c): domain of $(f \div g)$: $\{x \in \mathbf{R}\}$
1(d): domain of $(f \div g)$: $\{x \in \mathbf{R} \mid x > 2\}$
1(e): domain of $(f \div g)$: $\{x \in \mathbf{R}\}$
1(f): domain of $(f \div g)$: $\{x \in \mathbf{R} \mid x > 0\}$

3. a) $\frac{260}{1 + 24(0.9)^t} = \frac{260}{1 + 24(0.9)^{20}}$
$\doteq 66$ cm

The rate of change is $[66 - (260 \div 25)] \div 20$ or 2.798 cm/day.

b) The maximum height is 260, so half of 260 is 130 cm.

$$130 = \frac{260}{1 + 24(0.9)^t}$$
$$130 + 3120(0.9)^t = 260$$
$$3120(0.9)^t = 130$$
$$(0.9)^t = 130 \div 3120$$
$$t \log 0.9 = \log(130 \div 3120)$$
$$t \doteq 30 \text{ days}$$

c) $\left(\dfrac{260}{1 + 24(0.9)^{30.1}} - \dfrac{260}{1 + 24(0.9)^{30}}\right) \div 0.1$
$= 6.848$ cm/day

d) It slows down and eventually comes to zero. This is seen on the graph as it becomes horizontal at the top.

Mid-Chapter Review, p. 544

1. multiplication

2. a) $(f + g)(x) = \{(-9, -2 + 4), (-6, -3 + -6), (0, 2 + 12)\}$
$= \{(-9, 2), (-6, -9), (0, 14)\}$

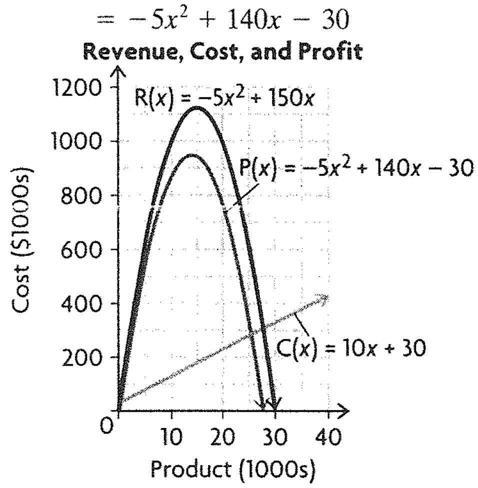
b) $(g + f)(x) = \{(-9, 4 + -2), (-6, -6 + -3), (0, 12 + 2)\}$
$= \{(-9, 2), (-6, -9), (0, 14)\}$

c) $(f - g)(x) = \{(-9, -2 - 4), (-6, -3 - -6), (0, 2 - 12)\}$
$= \{(-9, -6), (-6, 3), (0, -10)\}$

d) $(g - f)(x) = \{(-9, 4 - -2), (-6, -6 - -3), (0, 12 - 2)\}$
$= \{(-9, 6), (-6, -3), (0, 10)\}$

3. a) $P(x) = R(x) - C(x)$
$= -5x^2 + 150x - (10x + 30)$
$= -5x^2 + 140x - 30$

b)



c) $= -5x^2 + 140x - 30$
$= -5(7.5)^2 + 140(7.5) - 30$
$= \$738.75$ thousand
$= \$738\,750$

4. a) $R(h) = 24.39h$

b) $N(h) = 24.97h$

c) $W(h) = 24.78h$

d) $S(h) = 24.39h + 0.58h + 0.39h$
$= 25.36h$

e) $25.36(8) + 1.5(25.36)(3) = \$317$

5. a) $(f \times g)(x) = \left(x + \dfrac{1}{2}\right)\left(x + \dfrac{1}{2}\right)$
$= x^2 + x + \dfrac{1}{4}$

$D = \{x \in \mathbf{R}\}$

b) $(f \times g)(x) = \sin(3x)(\sqrt{x - 10})$
$D = \{x \in \mathbf{R} | x \geq 10\}$

c) $(f \times g)(x) = 11x^3 \times \dfrac{2}{x + 5}$
$= \dfrac{22x^3}{x + 5}$

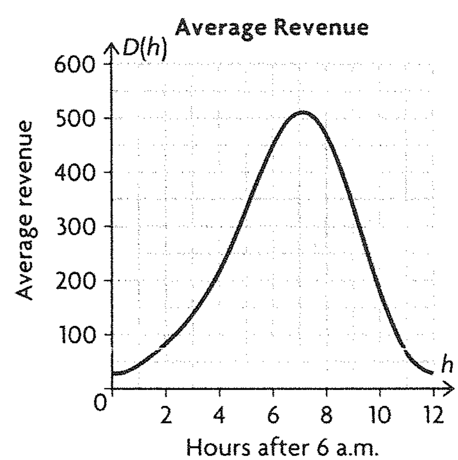
$D = \{x \in \mathbf{R} | x \neq -5\}$

d) $(f \times g)(x) = (90x - 1)(90x + 1)$
$= 8100x^2 - 1$

$D = \{x \in \mathbf{R}\}$

6. a) $C(h) \times D(h) = R(h)$
$= 90 \cos\left(\dfrac{\pi}{6}h\right) \sin\left(\dfrac{\pi}{6}h\right) - 102 \sin\left(\dfrac{\pi}{6}h\right)$
$- 210 \cos\left(\dfrac{\pi}{6}h\right) + 238$

b)



c) $R(2) = 90\cos\left(\frac{\pi}{6}(2)\right)\sin\left(\frac{\pi}{6}(2)\right)$
$- 102\sin\left(\frac{\pi}{6}(2)\right) - 210\cos\left(\frac{\pi}{6}(2)\right) + 238$
$\doteq \$470.30$

7. a) $(f \div g)(x) = 240 \div 3x$
$= \frac{80}{x}$
$D = \{x \in \mathbf{R} \mid x \neq 0\}$

b) $(f \div g)(x) = \frac{10x^2}{x^3 - 3x}$
$= \frac{10x^2}{x(x^2 - 3)}$
$= \frac{10x^2}{x^2 - 3}$
$D = \{x \in \mathbf{R} \mid x \neq \pm\sqrt{3}\}$

c) $(f \div g)(x) = \frac{x + 8}{\sqrt{x - 8}}$
$D = \{x \in \mathbf{R} \mid x > 8\}$

d) $(f \div g)(x) = \frac{7x^2}{\log x}$
$D = \{x \in \mathbf{R} \mid x > 0\}$

8. csc x, sec x, cot x

9.5 Composition of Functions, pp. 552–554

1. a) $g(0) = 1 - 0^2$
$= 1$
$f(g(0)) = f(1)$
$= 2(1) - 3$
$= 2 - 3$
$= -1$

b) $f(4) = 2(4) - 3$
$= 8 - 3$
$= 5$
$g(f(4)) = g(5)$
$= 1 - 5^2$
$= 1 - 25$
$= -24$

c) $g(-8) = 1 - (-8)^2$
$= 1 - 64$
$= -63$
$(f \circ g)(-8) = f(-63)$
$= 2(-63) - 3$
$= -126 - 3$
$= -129$

d) $g\left(\frac{1}{2}\right) = 1 - \left(\frac{1}{2}\right)^2$
$= 1 - \frac{1}{4}$
$= \frac{3}{4}$
$(g \circ g)\left(\frac{1}{2}\right) = g\left(\frac{3}{4}\right)$
$= 1 - \left(\frac{3}{4}\right)^2$
$= 1 - \frac{9}{16}$
$= \frac{7}{16}$

e) $(f \circ f^{-1})(1) = 1$

f) $g(2) = 1 - 2^2$
$= 1 - 4$
$= -3$
$(g \circ g)(2) = g(-3)$
$= 1 - (-3)^2$
$= 1 - 9$
$= -8$

2. a) $(g \circ f)(2) = g(5)$
$= 3$

b) $(f \circ f)(1) = f(2)$
$= 5$

c) $(f \circ g)(5) = f(3)$
$= 10$

d) $(f \circ g)(0)$ is undefined because there is no $g(0)$.

e) $(f \circ f^{-1})(2) = 2$

f) $(g^{-1} \circ f)(1) = g^{-1}(2)$
$= 4$

3. a) $f(g(2)) = f(5)$
$= 5$

b) $g(f(4)) = g(3)$
$= 5$

c) $(g \circ g)(-2) = g(1)$
$= 4$

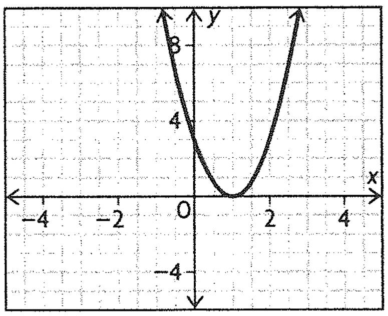
d) $(f \circ f)(2) = f(-1)$
$f(-1)$ does not exist, so $(f \circ f)(2)$ is undefined.

4. a) $d(5) = 80(5)$
$= 400$
$C(d(5)) = C(400)$
$= 0.09(400)$
$= 36$
It costs \$36 to travel for 5 hours.

b) $C(d(t))$ represents the relationship between the time driven and the cost of gasoline.

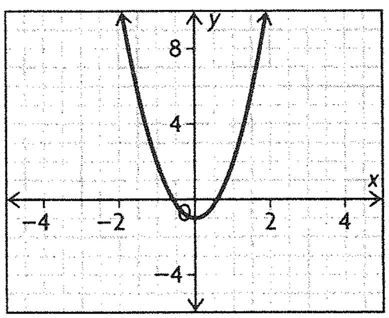
5. **a)** $f(g(x)) = f(x - 1)$
$= 3(x - 1)^2$
$= 3(x^2 - 2x + 1)$
$= 3x^2 - 6x + 3$

The domain is $\{x \in \mathbf{R}\}$.



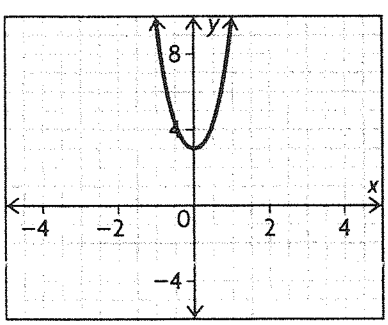
$g(f(x)) = g(3x^2)$
$= 3x^2 - 1$

The domain is $\{x \in \mathbf{R}\}$.



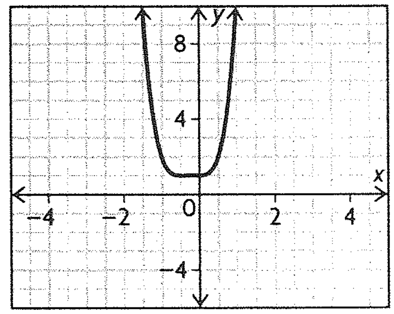
b) $f(g(x)) = f(x^2 + 1)$
$= 2(x^2 + 1)^2 + (x^2 + 1)$
$= 2(x^4 + 2x^2 + 1) + x^2 + 1$
$= 2x^4 + 4x^2 + 2 + x^2 + 1$
$= 2x^4 + 5x^2 + 3$

The domain is $\{x \in \mathbf{R}\}$.



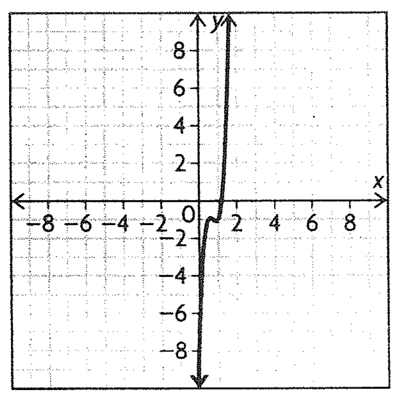
$g(f(x)) = g(2x^2 + x)$
$= (2x^2 + x)^2 + 1$
$= 4x^4 + 4x^3 + x^2 + 1$

The domain is $\{x \in \mathbf{R}\}$.



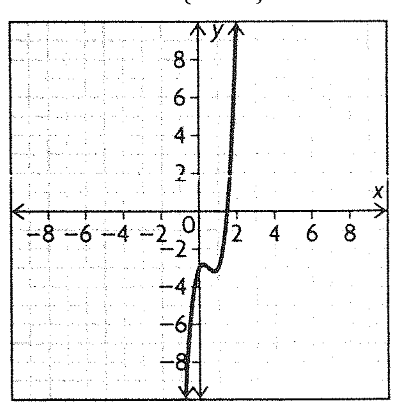
c) $f(g(x)) = f(2x - 1)$
$= 2(2x - 1)^3 - 3(2x - 1)^2 + (2x - 1) - 1$
$= 2(8x^3 - 12x^2 + 6x - 1) - 3(4x^2 - 4x + 1) + 2x - 1 - 1$
$= 16x^3 - 24x^2 + 12x - 2 - 12x^2 + 12x - 3 + 2x - 2$
$= 16x^3 - 36x^2 + 26x - 7$

The domain is $\{x \in \mathbf{R}\}$.



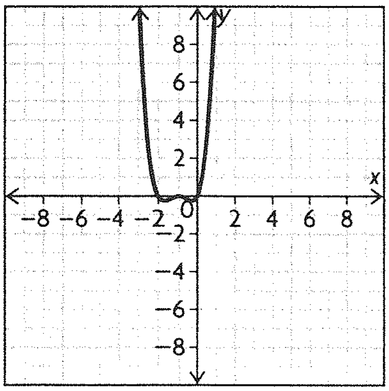
$g(f(x)) = g(2x^3 - 3x^2 + x - 1)$
$= 2(2x^3 - 3x^2 + x - 1) - 1$
$= 4x^3 - 6x^2 + 2x - 2 - 1$
$= 4x^3 - 6x^2 + 2x - 3$

The domain is $\{x \in \mathbf{R}\}$.



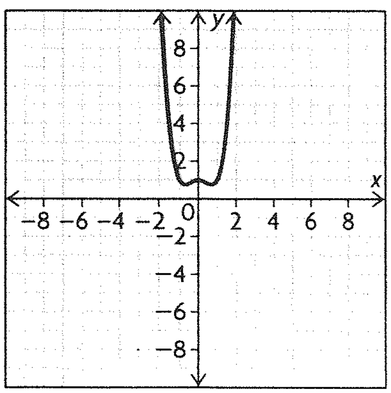
d) $f(g(x)) = f(x + 1)$
$= (x + 1)^4 - (x + 1)^2$
$= (x + 1)^2((x + 1)^2 - 1)$
$= (x^2 + 2x + 1)(x^2 + 2x + 1 - 1)$
$= (x^2 + 2x + 1)(x^2 + 2x)$
$= x^4 + 2x^3 + 2x^3 + 4x^2 + x^2 + 2x$
$= x^4 + 4x^3 + 5x^2 + 2x$

The domain is $\{x \in \mathbf{R}\}$.



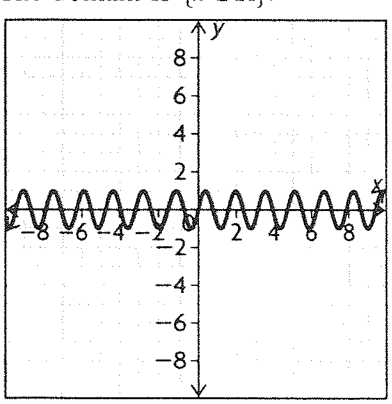
$g(f(x)) = g(x^4 - x^2)$
$= x^4 - x^2 + 1$

The domain is $\{x \in \mathbf{R}\}$.



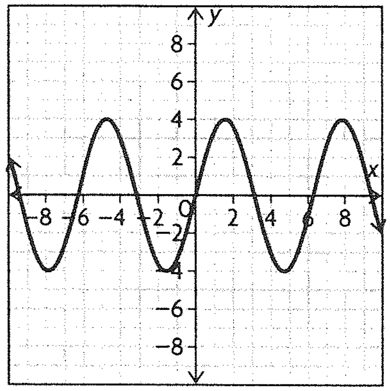
e) $f(g(x)) = f(4x)$
$= \sin 4x$

The domain is $\{x \in \mathbf{R}\}$.



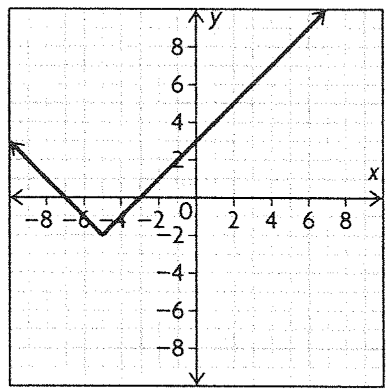
$g(f(x)) = g(\sin x)$
$= 4 \sin x$

The domain is $\{x \in \mathbf{R}\}$.



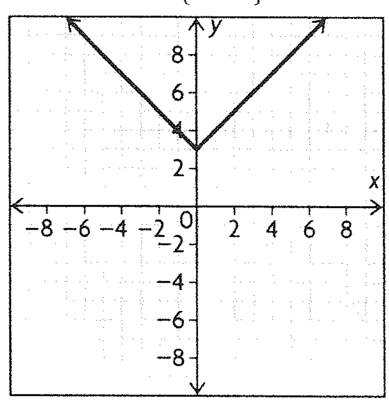
f) $f(g(x)) = f(x + 5)$
$= |x + 5| - 2$

The domain is $\{x \in \mathbf{R}\}$.



$g(f(x)) = g(|x| - 2)$
$= |x| - 2 + 5$
$= |x| + 3$

The domain is $\{x \in \mathbf{R}\}$.



6. a) $f \circ g = f(\sqrt{x - 4})$
$= 3\sqrt{x - 4}$

The domain is $\{x \in \mathbf{R} \mid x \geq 4\}$ and the range is $\{y \in \mathbf{R} \mid y \geq 0\}$.

$$g \circ f = g(3x)$$
$$= \sqrt{3x - 4}$$

The domain is $\{x \in \mathbf{R} | x \geq \frac{4}{3}\}$ and the range is $\{y \in \mathbf{R} | y \geq 0\}$.

b) $f \circ g = f(3x + 1)$
$$= \sqrt{3x + 1}$$

The domain is $\{x \in \mathbf{R} | x \geq -\frac{1}{3}\}$ and the range is $\{y \in \mathbf{R} | y \geq 0\}$.

$$g \circ f = g(\sqrt{x})$$
$$= 3\sqrt{x} + 1$$

The domain is $\{x \in \mathbf{R} | x \geq 0\}$ and the range is $\{y \in \mathbf{R} | y \geq 1\}$.

c) $f \circ g = f(x^2)$
$$= \sqrt{4 - (x^2)^2}$$
$$= \sqrt{4 - x^4}$$

The domain is $\{x \in \mathbf{R} | -\sqrt{2} \leq x \leq \sqrt{2}\}$ and the range is $\{y \in \mathbf{R} | y \geq 0\}$.

$$g \circ f = g(\sqrt{4 - x^2})$$
$$= (\sqrt{4 - x^2})^2$$
$$= 4 - x^2$$

The domain is $\{x \in \mathbf{R} | -2 \leq x \leq 2\}$ and the range is $\{y \in \mathbf{R} | 0 < y < 2\}$.

d) $f \circ g = f(\sqrt{x - 1})$
$$= 2\sqrt{x - 1}$$

The domain is $\{x \in \mathbf{R} | x \geq 1\}$ and the range is $\{y \in \mathbf{R} | y \geq 1\}$.

$$g \circ f = g(2^x)$$
$$= \sqrt{2^x - 1}$$

The domain is $\{x \in \mathbf{R} | x \geq 0\}$ and the range is $\{y \in \mathbf{R} | y \geq 0\}$.

e) $f \circ g = f(\log x)$
$$= 10^{\log x}$$
$$= x$$

The domain is $\{x \in \mathbf{R} | x > 0\}$ and the range is $\{y \in \mathbf{R}\}$.

$$g \circ f = g(10^x)$$
$$= \log 10^x$$
$$= x$$

The domain is $\{x \in \mathbf{R}\}$ and the range is $\{y \in \mathbf{R}\}$.

f) $f \circ g = f(5^{2x} + 1)$
$$= \sin(5^{2x} + 1)$$

The domain is $\{x \in \mathbf{R}\}$ and the range is $\{y \in \mathbf{R} | -1 \leq y \leq 1\}$.

$$g \circ f = g(\sin x)$$
$$= 5^{2 \sin x} + 1$$

The domain is $\{x \in \mathbf{R}\}$ and the range is $\{y \in \mathbf{R} | \frac{26}{25} \leq y \leq 26\}$.

7. a) Answers may vary. For example, $f(x) = \sqrt{x}$ and $g(x) = x^2 + 6$

b) Answers may vary. For example, $f(x) = x^6$ and $g(x) = 5x - 8$

c) Answers may vary. For example, $f(x) = 2^x$ and $g(x) = 6x + 7$

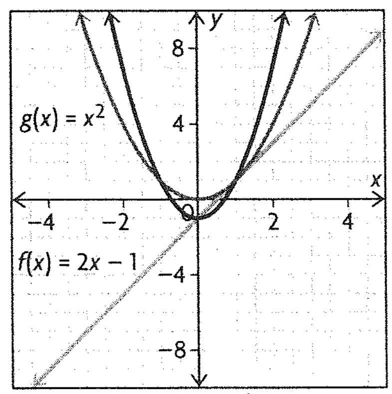
d) Answers may vary. For example, $f(x) = \frac{1}{x}$ and $g(x) = x^3 - 7x + 2$

e) Answers may vary. For example, $f(x) = \sin^2 x$ and $g(x) = 10x + 5$

f) Answers may vary. For example, $f(x) = \sqrt[3]{x}$ and $g(x) = (x + 4)^2$

8. a) $(f \circ g)(x) = f(x^2)$
$$= 2x^2 - 1$$

b)



$(f \circ g)(x) = 2x^2 - 1$

c) It is compressed by a factor of 2 and translated down 1 unit.

9. a) $f(g(x)) = f(3x + 2)$
$$= 2(3x + 2) - 1$$
$$= 6x + 4 - 1$$
$$= 6x + 3$$

The slope of $g(x)$ has been multiplied by 2, and the y-intercept of $g(x)$ has been vertically translated 1 unit up.

b) $g(f(x)) = g(2x - 1)$
$$= 3(2x - 1) + 2$$
$$= 6x - 3 + 2$$
$$= 6x - 1$$

The slope of $f(x)$ has been multiplied by 3.

10. $D(p) = 0.80(975 + 39.95p)$
$$= 780 + 31.96p$$

11. $f(g(x)) = f(0.75x)$
$$= 0.08(0.75x)$$
$$= 0.06x$$

12. a) $d(s) = \sqrt{16 + s^2}$; $s(t) = 560t$

b) $d(s(t)) = \sqrt{16 + 313\,600t^2}$, where t is the time in hours and $d(s(t))$ is the distance in kilometres

13. $c(v(t)) = \left(\frac{40 + 3t + t^2}{500} - 0.1\right)^2 + 0.15;$
The car is running most economically 2 hours into the trip.

14. Graph A(k); $f(x)$ is vertically compressed by a factor of 0.5 and reflected in the x-axis.
Graph B(b); $f(x)$ is translated 3 units to the right.
Graph C(d); $f(x)$ is horizontally compressed by a factor of $\frac{1}{2}$.
Graph D(1); $f(x)$ is translated 4 units down.
Graph E(g); $f(x)$ is translated 3 units up.
Graph F(c); $f(x)$ is reflected in the y-axis.

15. Sum: $y = f + g$
$f(x) = \frac{4}{x - 3}; g(x) = 1$
Product: $y = f \times g$
$f(x) = x - 3;$
$g(x) = \frac{x + 1}{(x - 3)^2}$
Quotient: $y = f \div g$
$f(x) = 1 + x; g(x) = x - 3$
Composition: $y = f \circ g$
$f(x) = \frac{4}{x} + 1; g(x) = x - 3$

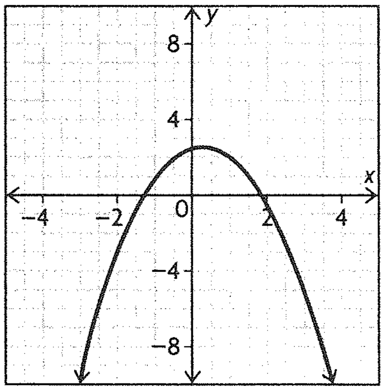
16. a)
$$\begin{aligned} f(k) &= f(x(t(k))) \\ &= f(x(3k - 2)) \\ &= f(3(3k - 2) + 2) \\ &= f(9k - 6 + 2) \\ &= f(9k - 4) \\ &= 3(9k - 4) - 2 \\ &= 27k - 12 - 2 \\ &= 27k - 14 \end{aligned}$$
b)
$$\begin{aligned} f(k) &= f(x(t(k))) \\ &= f(x(3k - 5)) \\ &= f\left(\sqrt{3(3k - 5) - 1}\right) \\ &= f\left(\sqrt{9k - 15 - 1}\right) \\ &= f\left(\sqrt{9k - 16}\right) \\ &= 2\sqrt{9k - 16} - 5 \end{aligned}$$

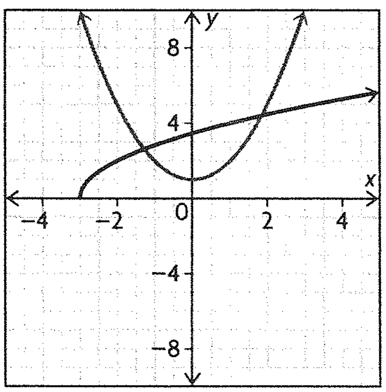
9.6 Techniques for Solving Equations and Inequalities, pp. 560–562

1. Use the graph to find the solutions.
a) i) $x = \frac{1}{2}, 2, \text{ or } \frac{7}{2}$
ii) $x = -1$ or 2
b) i) $\frac{1}{2} < x < 2$ or $x > \frac{7}{2}$
ii) $-1 < x < 2$
c) i) $x \le \frac{1}{2}; 2 \le x \le \frac{7}{2}$
ii) $x \le -1$ or $x \ge 2$
d) i) $\frac{1}{2} \le x \le 2$ or $x \ge \frac{7}{2}$
ii) $-1 \le x \le 2$

2. a) $3 = 2^{2x}$
Try $x = 1$: $3 = 2^2$
$3 = 4$ Too high
Try $x = 0.5$: $3 = 2^1$
$3 = 2$ Too low
Try 0.6: $3 = 2^{1.2}$
$3 = 2.3$ Too low
Try 0.8: $3 = 2^{1.6}$
$3 = 3.03$
So, $x \doteq 0.8$
b) $0 = \sin(0.25x^2)$
Try $x = 0$: $0 = \sin 0$
$0 = 0$ Correct
Try $x = 2$: $0 = \sin(0.25(4))$
$0 = 0.84$ Too high
Try $x = 3$: $0 = \sin(0.25(9))$
$0 = 0.78$ Too high
Try $x = 3.5$: $0 = \sin(0.25(12.25))$
$0 = 0.08$ Close
Try $x = 3.6$: $0 = \sin(0.25(12.96))$
$0 = -0.1$
So, $x = 0$ and 3.5
c) $3x = 0.5x^3$
Try $x = -2$: $3(-2) = 0.5(-2)^3$
$-6 = -4$
Try $x = -3$: $3(-3) = 0.5(-3)^3$
$-9 = -13.5$
Try $x = -2.5$: $3(-2.5) = 0.5(-2.5)^3$
$-7.5 = -7.8$
Try $x = -2.4$: $3(-2.4) = 0.5(-2.4)^3$
$-7.2 = -6.9$
So, $x \doteq -2.4$
d) $\cos x = x$
Try $x = 0$: $\cos 0 = 0$
$1 = 0$
Try 0.5: $\cos 0.5 = 0.5$
$0.8 = 0.5$
Try 0.6: $\cos 0.6 = 0.6$
$0.8 = 0.6$

Try 0.7: $\cos 0.7 = 0.7$
$0.76 = 0.7$
So, $x \doteq 0.7$
3. Graph and use the graph to find the solutions.
$x = -1.3$ or 1.8





4. Use the graph to find the solutions.
$f(x) < g(x)$: $1.3 < x < 1.6$
$f(x) = g(x)$: $x = 0$ or 1.3
$f(x) > g(x)$: $0 < x < 1.3$ or $1.6 < x < 3$
5. a) $5 \sec x = -x^2$
Try $x = 2$: $5 \sec 2 = -2^2$
$-12.0 = -4$
Try $x = 2.5$: $5 \sec 2.5 = -2.5^2$
$-6.24 = -6.25$
So, $x \doteq 2.5$
b) $\sin^3 x = \sqrt{x} - 1$
Try $x = 2$: $\sin^3 2 = \sqrt{2} - 1$
$0.75 = 0.41$
Try $x = 2.2$: $\sin^3 2.2 = \sqrt{2.2} - 1$
$0.53 = 0.48$
So, $x \doteq 2.2$
c) $5^x = x^5$
Try $x = 1$: $5^1 = 1^5$
$5 = 1$
Try $x = 2$:
$5^2 = 2^5$
$25 = 32$
Try $x = 1.9$: $5^{1.9} = 1.9^5$
$21.28 = 24.76$
Try $x = 1.8$: $5^{1.8} = 1.8^5$
$18.12 = 18.90$
So, $x \doteq 1.8$
d) $\cos x = \frac{1}{x}$

Try $x = -2$: $\cos -2 = \frac{1}{-2}$
$-0.42 = -0.5$
Try $x = -2.1$: $\cos -2.1 = \frac{1}{-2.1}$
$-0.50 = -0.58$
So, $x \doteq -2.1$
e) $\log x = (x - 10)^2 + 1$
Try $x = 9$: $\log 9 = (9 - 10)^2 + 1$
$0.95 = 0$
Try $x = 10$: $\log 10 = (10 - 10)^2 + 1$
$1 = 1$
So, $x = 10$.
f) $\sin(2\pi x) = -4x^2 + 16x - 12$
Try $x = 0$: $\sin 0 = -4(0)^2 + 16(0) - 12$
$0 = -12$
Try $x = 1$: $\sin 2\pi = -4(1)^2 + 16(1) - 12$
$0 = 0$
Try $x = 3$: $\sin 6\pi = -4(3)^2 + 16(3) - 12$
$0 = 0$
So, $x = 1$ or 3
6. Use a graphing calculator to estimate the solutions.
a) $x = -1.81$ or 0.48
b) $x = -1.38$ or 1.6
c) $x = -1.38$ or 1.30
d) $x = -0.8, 0,$ or 0.8
e) $x = 0.21$ or 0.74
f) $x = 0, 0.18, 0.38,$ or 1
7. Since the graph crosses the x-axis at $x = 0.7$, the x-coordinate of the solution is 0.7. Use $x = 0.7$ to find the y-coordinate.
$y = -3x^2$
$y = -3(0.7)^2$
$y = -1.47$
So, the coordinates are $(0.7, -1.5)$.
8. $2.3(0.96)^t = 1.95(0.97)^t$
Use a graphing calculator to estimate the solution.
$t \doteq 15$
So, they will be about the same in 1997 + 15 or 2012.

9. Use a graphing calculator to estimate the solutions.
a) $x \in (-0.57, 1)$
b) $x \in [0, 0.58]$
c) $x \in (-\infty, 0)$
d) $x \in (0.17, 0.83)$
e) $x \in (0.35, 1.51)$
f) $x \in (0.1, 0.5)$
10. Answers may vary. For example, $f(x) = x^3 + 5x^2 + 2x - 8$ and $g(x) = 0$
11. Answers may vary. For example, $f(x) = -x^2 + 25$ and $g(x) = -x + 5$
12.
$$\begin{aligned} a\cos x &= bx^3 + 6 \\ a\cos(-1.2) &= b(-1.2)^3 + 6 \\ a\cos(-0.7) &= b(-0.7)^3 + 6 \\ 0.36a &= -1.728b + 6 \\ 0.76a &= -0.343b + 6 \\ \frac{-1.728b + 6}{0.36} &= \frac{-0.343b + 6}{0.76} \\ -1.31328b + 4.56 &= -0.12348b + 2.16 \\ -1.1898b &= -2.4 \\ b &\doteq 2 \\ \text{So, } a &= \frac{-1.728(2) + 6}{0.36} \\ &\doteq 7 \end{aligned}$$

13. Answers may vary. For example:

Perform the necessary algebraic operations to move all of the terms on the right side of the equation to the left side of the equation.

↓

Construct the function $f(x)$, such that $f(x)$ equals the left side of the equation.

↓

Graph the function $f(x)$.

↓

Determine the x-intercepts of the graph that fall within the interval provided, if applicable.

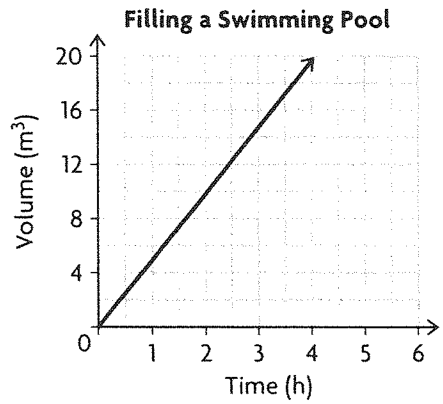
↓

The x-intercepts of the graph are the solutions to the equation.

14. Use a graphing calculator to determine the solutions.
$x = 0 \pm 2n$, $x = -0.67 \pm 2n$ or $x = 0.62 \pm 2n$, where $n \in \mathbf{I}$
15. Use a graphing calculator to determine the solutions.
$x \in (2n, 2n + 1)$, where $n \in \mathbf{I}$

9.7 Modelling with Functions, pp. 569–574

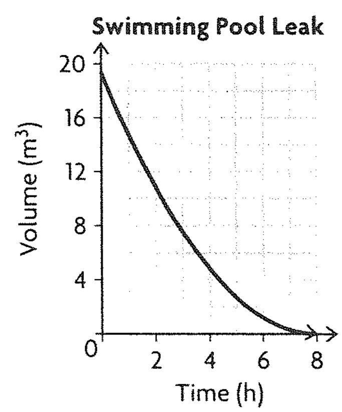
1. a) The equation of the graph is $6.25\pi\left(\frac{x}{4}\right)$



b) $y = 6.25\pi\left(\frac{x}{4}\right)$

c) $8 = 6.25\pi\left(\frac{x}{4}\right)$
$8 = 1.5625\pi x$
$1.6 = x$
So, it will take about 1.6 hours.

2. a) $y = \frac{6.25\pi}{64}(x - 8)^2$



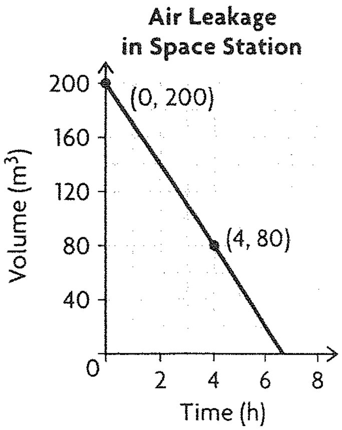
b) $V(t) = \frac{6.25\pi}{64}(t - 8)^2$

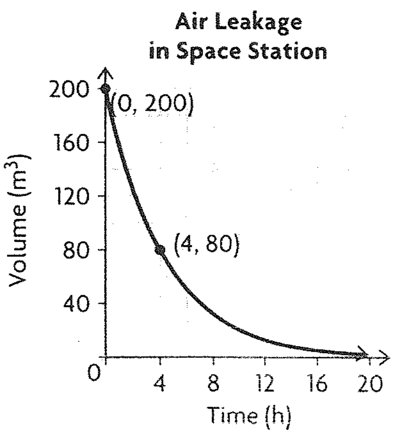
c)
$$\begin{aligned} V(2) &= \frac{6.25\pi}{64}(2 - 8)^2 \\ &= \frac{6.25\pi}{64}(-6)^2 \\ &= \frac{6.25\pi}{64}(36) \\ &\doteq 11 \text{ m}^3 \end{aligned}$$

d) The initial volume is 19.6 m^3.
So, the rate of change is $(11 - 19.6) \div 2$ or -4.3 m^3/hr

e) As time elapses, the pool is losing less water in the same amount of time.

3. a) Answers may vary. For example:





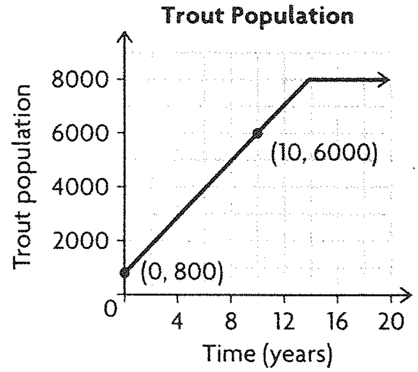
b) From the linear graph, the y-intercept is $(0, 200)$ and the slope is -30, so the equation is

$$V(t) = -30t + 200$$
$$V(t) = -30t + 200$$
$$0 = -30t + 200$$
$$-200 = -30t$$
$$6.7 \doteq t$$

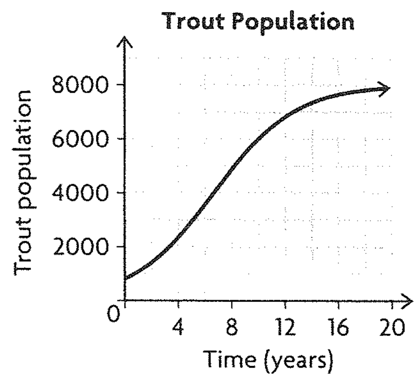
c) Using a graphing calculator, the equation that fits the model is $V(t) = 200(0.795)^t$

$$V(t) = 200(0.795)^t$$
$$20 = 200(0.795)^t$$
$$0.1 = 0.795^t$$
$$\log 0.1 = t \log 0.795$$
$$10 \doteq t$$

4. a)



b) $P(t) = \dfrac{8000}{1 + 9(0.719)^t}$



c) $P(4) = \frac{8000}{1 + 9(0.719)^4} \doteq 2349$ trout four years after restocking

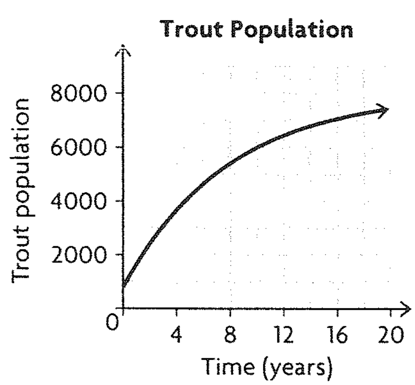
d) The rate of change is $(2349 - 800) \div 4$ or 387.25 trout per year.

5. a) the carrying capacity of the lake; 8000

b) Use $(0, 800)$ and $(10, 6000)$.

$$800 = 8000 - a(b)^0$$
$$800 = 8000 - a$$
$$-7200 = -a$$
$$7200 = a$$
$$6000 = 8000 - 7200(b)^{10}$$
$$-2000 = -7200(b)^{10}$$
$$0.278 = b^{10}$$
$$0.278^{\frac{1}{10}} = b$$
$$0.88 \doteq b$$

c)



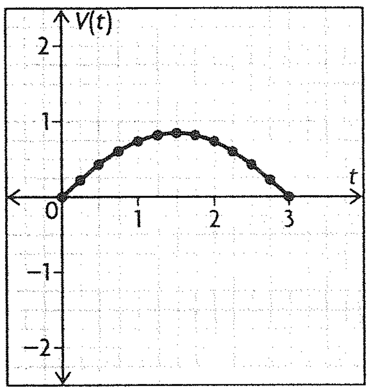
d) $P(4) = 8000 - 7200(0.88)^4$
$= 8000 - 4317.81$
$\doteq 3682$

e) The rate of change is $(3682 - 800) \div 4$ or 720.5 trout per year.

f) In the model in the previous problem, the carrying capacity of the lake is divided by a number that gets smaller and smaller, while in this model, a number that gets smaller and smaller is subtracted from the carrying capacity of the lake.

6. Answers may vary. For example, the first model more accurately calculates the current price of gasoline because prices are rising quickly.

7. a) $V(t) = 0.85\cos\left(\frac{\pi}{3}(t - 1.5)\right)$



b) The scatter plot and the graph are very close to being the same, but they are not exactly the same.

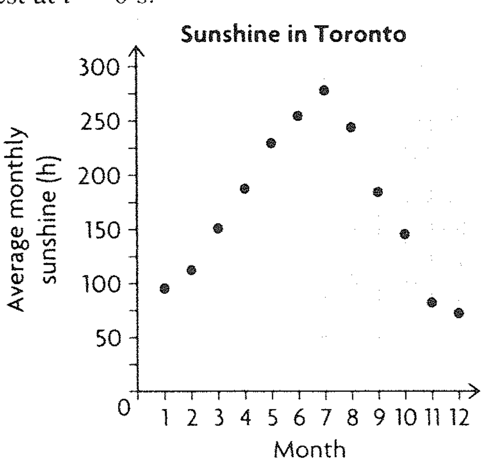
c) $V(6) = 0.85\cos\left(\frac{\pi}{3}(6 - 1.5)\right)$
$= 0.85\cos\left(\frac{\pi}{3}(4.5)\right)$
$= 0$ L/s

d) From the graph, the rate of change appears to be at its smallest at $t = 1.5$ s.

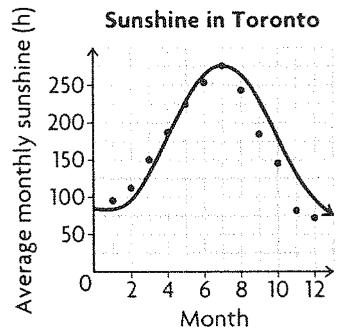
e) It is the maximum of the function.

f) From the graph, the rate of change appears to be greatest at $t = 0$ s.

8. a)



b)



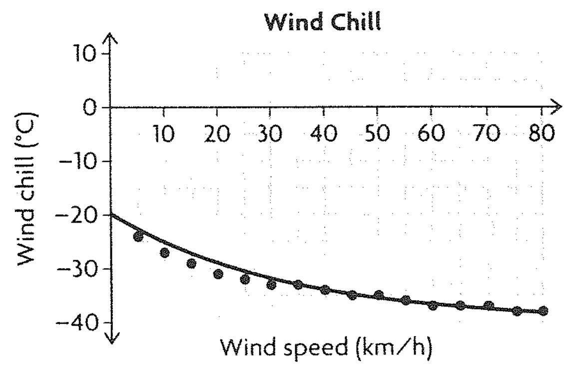
c) The equation is

$S(t) + -97\cos\left(\frac{\pi}{6}(t - 1)\right) + 181.$

d) From the model, the maximum will be at $t = 7$ and the minimum will be at $t = 1$.

e) It doesn't fit it perfectly, because, actually, the minimum is not at $t = 1$, but at $t = 12$.

9. a)



b) Answers may vary. For example,
$C(s) = -38 + 14(0.97)^s$

c) $C(0) = -38 + 14(0.97)^0$
$= -38 + 14$
$= -24°C$

$C(100) = -38 + 14(0.97)^{100}$
$= -38 + 0.666$
$\doteq -37.3°C$

$C(200) = -38 + 14(0.97)^{200}$
$= -38 + 0.032$
$\doteq -38°C$

These answers don't appear to be very reasonable, because the wind chill for a wind speed of 0 km/h should be −20°C, while the wind chills for wind speeds of 100 km/h and 200 km/h should be less than −38°C. The model only appears to be somewhat accurate for wind speeds of 10 to 70 km/hr.

10. a) Answers will vary; for example, one polynomial model is $P(t) = 1.4t^2 + 3230$, while an exponential model is $P(t) = 3230(1.016)^t$. While neither model is perfect, it appears that the polynomial model fits the data better.

b) $P(155) = 1.4(155)^2 + 3230$
$\doteq 36\,865$
$P(155) = 3230(1.016)^{155}$
$\doteq 37\,820$

c) A case could be made for either model. The polynomial model appears to fit the data better, but population growth is usually exponential.

d) According to the polynomial model, in 2000, the population was increasing at a rate of about 389 000 per year, while according to the exponential model, in 2000, the population was increasing at a rate of about 465 000 per year.

11. a) $P(t) = 3339.18(1.132\,25)^t$

b) $75 = 3339.18(1.132\,25)^t$
Use a graphing calculator to solve.
$t \doteq -31$
So, they were introduced around the year 1924.

c) rate of growth
$= (93\,100 - 650) \div (35 - 0)$
$\doteq 2641$ rabbits per year.

d) $P(65) = 3339.18(1.132\,25)^{65}$
$\doteq 10\,712\,509.96$

12. The amplitude is 155.6. The equation of the axis is $y = 0$. The period is $\frac{1}{60}$ s.

a) $V(t) = 155.6 \sin\left(120\pi t + \dfrac{\pi}{2}\right)$

b) $V(t) = 155.6 \cos(120\pi t)$

c) The cosine function was easier to determine. The cosine function is at its maximum when the argument is 0, so no horizontal translation was necessary.

13. a) Answers will vary; for example, a linear model is $P(t) = -9t + 400$; a quadratic model is $P(t) = \frac{23}{90}(t - 30)^2 + 170$; an exponential model is $P(t) = 400(0.972)^t$.
The exponential model fits the data far better than the other two models.

b) $P(60) = -9(60) + 400$
$= -540 + 400$
$= -140$ kPa

$$P(60) = \frac{23}{90}(60 - 30)^2 + 170$$
$$= \frac{23}{90}(30)^2 + 170$$
$$= \frac{23}{90}(900) + 170$$
$$= 230 + 170$$
$$= 400 \text{ kPa}$$

$P(60) = 400(0.972)^{60}$
$\doteq 73$ kPa

c) The exponential model gives the most realistic answer, because it fits the data the best. Also, the pressure must be less than 170 kPa, but it cannot be negative.

14. As a population procreates, the population becomes larger, and thus, more and more organisms exist that can procreate some more. In other words, the act of procreating enables even more procreating in the future.

15. a) linear, quadratic, or exponential
b) linear or quadratic
c) exponential

16. a) $T(n) = \dfrac{1}{6}n^3 + \dfrac{1}{2}n^2 + \dfrac{1}{3}n$

b) $47\,850 = \dfrac{1}{6}n^3 + \dfrac{1}{2}n^2 + \dfrac{1}{3}n$

So, $n \doteq 64.975$. So, it is not a tetrahedral number because n must be an integer.

17. a) Using a graphing calculator, the equation is $P(t) = 30.75(1.008\,418)^t$

b) In 2000, the growth rate of Canada was less than the growth rate of Ontario and Alberta.

Chapter Review, pp. 576–577

1. The only operation that will result in both vertical and horizontal asymptotes is division.

2. a) Since shop 1 contains $-1.4t^2$, it sales are decreasing after 2000. Since shop 2 contains all addition, its sales are increasing after 2000.

b) $S_{1+2} = 700 - 1.4t^2 + t^3 + 3t^2 + 500$
$= t^3 + 1.6t^2 + 1200$

c) $t = 6$: $t^3 + 1.6t^2 + 1200$
$= (6)^3 + 1.6(6)^2 + 1200$
$= 216 + 1.6(36) + 1200$
$= 216 + 57.6 + 1200$
$= 1473.6$ thousand or 1 473 600

d) The owner should close the first shop, because the sales are decreasing and will eventually reach zero.

3. a) $C(x) = 9.45x + 52\,000$

b) $I(x) = 15.8x$

c) $P(x) = I - C$
$= 15.8x - (9.45x + 52\,000)$
$= 15.8x - 9.45x - 52\,000$
$= 6.35x - 52\,000$

4. a) $(f \times g)(x) = 3\tan(7x) \times 4\cos(7x)$
$= 12\sin(7x)$

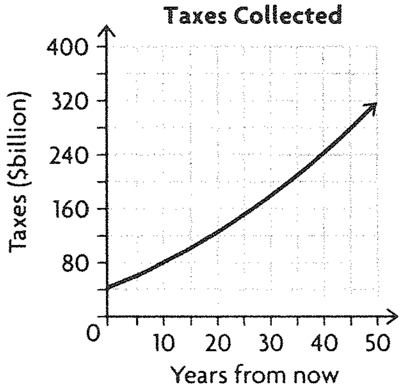
b) $(f \times g)(x) = \sqrt{3x^2} \times 3\sqrt{3x^2}$
$= 3(3x^2)$
$= 9x^2$

c) $(f \times g)(x) = (11x - 7)(11x + 7)$
$= 121x^2 - 49$

d) $(f \times g)(x) = ab^x \times 2ab^{2x}$
$= 2a^2b^{3x}$

5. a) $C \times A = (15\,000\,000(1.01)^t)(2850 + 200t)$
$= 42\,750\,000\,000(1.01)^t$
$+ 3\,000\,000\,000t(1.01)^t$

b)



d) $42\,750\,000\,000(1.01)^{26} + 3\,000\,000\,000t(1.01)^{26}$
$\doteq \$156\,402\,200\,032.31$

6. a) $(f \div g)(x) = 105x^3 \div 5x^4$
$= \dfrac{21}{x}$

b) $(f \div g)(x) = (x - 4) \div (2x^2 + x - 36)$
$= (x - 4) \div (x - 4)(2x + 9)$
$= \dfrac{1}{2x + 9}$

c) $(f \div g)(x) = \sqrt{x + 15} \div (x + 15)$
$= \dfrac{\sqrt{x + 15}}{x + 15}$

d) $(f \div g)(x) = 11x^5 \div 22x^2 \log x$
$= \dfrac{x^3}{2 \log x}$

7. a) $\{x \in \mathbf{R} \mid x \neq 0\}$

b) $\left\{x \in \mathbf{R} \,\middle|\, x \neq 4, x \neq -\dfrac{9}{2}\right\}$

c) $\{x \in \mathbf{R} \mid x > -15\}$

d) $\{x \in \mathbf{R} \mid x > 0\}$

8. a) Domain of $f(x)$: $\{x \in \mathbf{R} \mid x > -1\}$
Range of $f(x)$: $\{y \in \mathbf{R} \mid y > 0\}$
Domain of $g(x)$: $\{x \in \mathbf{R}\}$
Range of $g(x)$: $\{y \in \mathbf{R} \mid y \geq 3\}$

b) $f(g(x)) = f(x^2 + 3)$
$= \dfrac{1}{\sqrt{x^2 + 3 + 1}}$
$= \dfrac{1}{\sqrt{x^2 + 4}}$

c) $g(f(x)) = g\left(\dfrac{1}{\sqrt{x + 1}}\right)$
$= \left(\dfrac{1}{\sqrt{x + 1}}\right)^2 + 3$
$= \dfrac{1}{x + 1} + 3$
$= \dfrac{1}{x + 1} + \dfrac{3x + 3}{x + 1}$
$= \dfrac{3x + 4}{x + 1}$

d) $f(g(0)) = \dfrac{1}{\sqrt{0^2 + 4}}$
$= \dfrac{1}{\sqrt{4}}$
$= \dfrac{1}{2}$

e) $g(f(0)) = \dfrac{3(0) + 4}{0 + 1}$
$= \dfrac{4}{1}$
$= 4$

f) For $f(g(x))$: $\{x \in \mathbf{R}\}$
For $g(f(x))$: $\{x \in \mathbf{R} \mid x > -1\}$

9. a) $(f \circ f)(x) = f(x - 3)$
$= x - 3 - 3$
$= x - 6$

b) $(f \circ f \circ f)(x) = f(x - 6)$
$= x - 6 - 3$
$= x - 9$

c) $(f \circ f \circ f \circ f)(x) = f(x - 9)$
$= x - 9 - 3$
$= x - 12$

d) f composed with itself n times $= x - 3(1 + n)$

10. a) $A(r) = \pi r^2$

b) $r(C) = \dfrac{C}{2\pi}$

c) $A(r(C)) = A\left(\dfrac{C}{2\pi}\right)$
$= \pi\left(\dfrac{C}{2\pi}\right)^2$

$= \pi\left(\frac{C^2}{4\pi^2}\right)$

$= \frac{C^2}{4\pi}$

d) $\frac{C^2}{4\pi} = \frac{(3.6)^2}{4\pi}$

$\doteq 1.03$ m

11. Use the graph to find the solutions.
$f(x) < g(x)$: $-1.2 < x < 0$ or $x > 1.2$
$f(x) = g(x)$: $x = -1.2, 0,$ or 1.2
$f(x) > g(x)$: $x < -1.2$ or $0 < x < 1.2$

12. a) $-3 \csc x = x$
Try $x = 3$: $-3 \csc 3 = 3$
$-21.3 = 3$
Try $x = 4$: $-3 \csc 4 = 4$
$3.9 = 4$
So, $x \doteq 4.0$

b) $\cos^2 x = 3 - 2\sqrt{x}$
Try $x = 1$: $\cos^2 1 = 3 - 2\sqrt{1}$
$0.29 = 3 - 2(1)$
$0.29 = 1$
Try $x = 2$: $\cos^2 2 = 3 - 2\sqrt{2}$
$0.17 = 0.17$
So, $x \doteq 2.0$

c) $8^x = x^8$
Try $x = -0.7$: $8^{-0.7} = (-0.7)^8$
$0.23 = 0.06$
Try $x = -0.8$: $8^{-0.8} = (-0.8)^8$
$0.19 = 0.17$
So, $x \doteq -0.8$.

d) $7 \sin x = \frac{3}{x}$

Try $x = 0.6$: $7 \sin(0.6) = \frac{3}{0.6}$

$3.95 = 5$

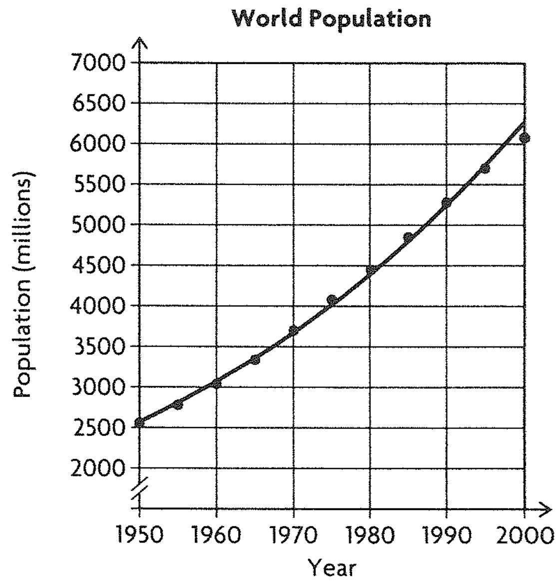
Try $x = 0.7$: $7 \sin(0.7) = \frac{3}{0.7}$

$4.5 = 4.3$
So, $x \doteq 0.7$.

13. a) The rate of change is
$(3200 - 2000) \div (7 - 5)$ or
600 frogs per year. So, the equation is:
$P(t) = 600t - 1000$
The slope is the rate at which the population is changing.

b) $P(t) = 617.6(1.26)^t$
617.6 is the initial population and 1.26 represents the growth.

14. Use a graphing calculator to graph the points and the model, $P(t) = 2570.99(1.018)^t$.



Use the graph to estimate the values for $t = 13, 23,$ and 90.
When $t = 13$, $P(t) = 3242$.
When $t = 23$, $P(t) = 3875$.
When $t = 90$, $P(t) = 12\,806$.

Chapter Self-Test, p. 578

1. a) $A(r) = 4\pi r^2$

b) $V = \frac{4}{3}\pi r^3$

$\frac{3V}{4\pi} = r^3$

$\sqrt[3]{\frac{3V}{4\pi}} = r$

So, $r(V) = \sqrt[3]{\frac{3V}{4\pi}}$

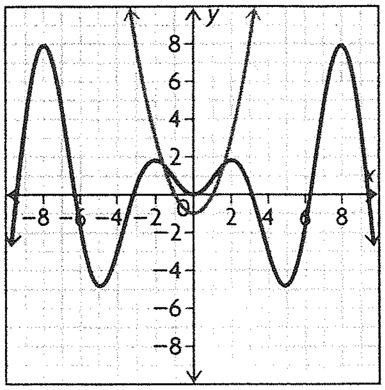
c) $A(r(V)) = A\left(\sqrt[3]{\frac{3V}{4\pi}}\right)$

$= 4\pi\left(\sqrt[3]{\frac{3V}{4\pi}}\right)^2$

$= 4\pi\left(\frac{3V}{4\pi}\right)^{\frac{2}{3}}$

d) $4\pi\left(\frac{3(0.75)}{4\pi}\right)^{\frac{2}{3}} \doteq 4 \text{ m}^2$

2. Draw the graph of each function and use it to determine when $x \sin x \geq x^2 - 1$.



From the graph, the solution is $-1.62 \leq x \leq 1.62$.

3. Answers may vary. For example, $g(x) = x^7$ and $h(x) = 2x + 3$

$g(x) = (x + 3)^7$ and $h(x) = 2x$

4. a) Use a graphing calculator to determine the regression equation.

$N(n) = 1n^3 + 8n^2 + 40n + 400$

b) $N(3) = 1(3)^3 + 8(3)^2 + 40(3) + 400$

$= 27 + 72 + 120 + 400$

$= 619$

5. $f(x) = 6x + b$

$-3 = 6(2) + b$

$-3 = 12 + b$

$-15 = b$

So, $f(x) = 6x - 15$

$$\begin{aligned} g(x) &= 5(x + 8)^2 - 1 \\ (f \times g)(x) &= (6x - 15)(5(x + 8)^2 - 1) \\ &= (6x - 15)(5(x^2 + 16x + 64) - 1) \\ &= (6x - 15)(5x^2 + 80x + 320 - 1) \\ &= (6x - 15)(5x^2 + 80x + 319) \\ &= 30x^3 + 405x^2 + 714x - 4785 \end{aligned}$$

6. a) There is a horizontal asymptote of $y = 275$ cm. This is the maximum height this species will reach.

b) $$150 = \frac{275}{1 + 26(0.85)^t}$$

$$\begin{aligned} 150(1 + 26(0.85)^t) &= 275 \\ 26(0.85)^t &= (275 \div 150) - 1 \\ 26(0.85)^t &\doteq 0.8333 \\ (0.85)^t &\doteq 0.032\,05 \\ t \log 0.85 &= \log 0.032\,05 \\ t &\doteq 21.2 \text{ months} \end{aligned}$$

7. Find when $C(x) = R(x)$

$5x + 18 = 2x^2$

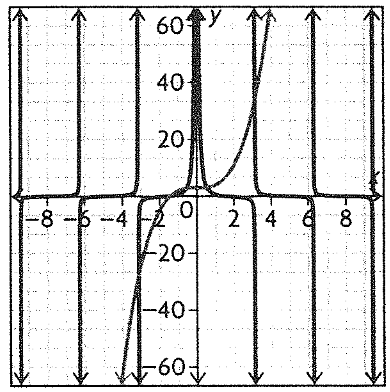
$0 = 2x^2 - 5x - 18$

$0 = (2x - 9)(x + 2)$

$x = 4.5$ or -2

Negative answers do not make sense in this context, so the answer is $x = 4.5$ or 4500 items.

8. Graph both sides on the equation and use the graph to find the solutions.



The solutions are $x = -3.1, -1.4, -0.6, 0.5$, or 3.2.

9. Division will turn it into a tangent function that is not sinusoidal.

Chapters 7–9: Cumulative Review, pp. 580–583

1. Since $\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$,

$$\begin{aligned} \sin\frac{2\pi}{5} &= \cos\left(\frac{2\pi}{5} - \frac{\pi}{2}\right) \\ &= \cos\left(\frac{4\pi}{10} - \frac{5\pi}{10}\right) \\ &= \cos\left(-\frac{\pi}{10}\right). \end{aligned}$$

Since $\cos\theta = \cos(2\pi - \theta)$,

$$\begin{aligned} \cos\left(-\frac{\pi}{10}\right) &= \cos\left(2\pi - \left(-\frac{\pi}{10}\right)\right) \\ &= \cos\left(2\pi + \frac{\pi}{10}\right). \end{aligned}$$

Since the period of the cosine function is 2π, $\cos\left(2\pi + \frac{\pi}{10}\right) = \cos\frac{\pi}{10}$. Therefore, answer (a) is correct. Also, since $\cos(\pi - \theta) = -\cos\theta$,

$$\begin{aligned} \cos\frac{\pi}{10} &= -\cos\left(\pi - \frac{\pi}{10}\right) \\ &= -\cos\left(\frac{10\pi}{10} - \frac{\pi}{10}\right) \\ &= -\cos\frac{9\pi}{10}. \end{aligned}$$

Therefore, answer (c) is correct. Also, since $\sin(\pi - \theta) = \sin\theta$,

$$\sin\frac{2\pi}{5} = \sin\left(\pi - \frac{2\pi}{5}\right)$$
$$= \sin\left(\frac{5\pi}{5} - \frac{2\pi}{5}\right)$$
$$= \sin\frac{3\pi}{5}.$$

Therefore, answer (b) is correct. Since answers (a), (b), and (c) are all correct, the correct answer is **d)**.

2. Since $\cos(a - b) = \cos a\cos b + \sin a\sin b$,

$$\cos\frac{\pi}{12} = \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$$
$$= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$
$$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$
$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$
$$= \frac{\sqrt{2} + \sqrt{6}}{4}.$$

Therefore, the correct answer is **b)**.

3. Since $\sin\alpha = \frac{12}{13}$, the leg opposite the angle α in a right triangle has a length of 12, while the hypotenuse of the right triangle has a length of 13. For this reason, the other leg of the right triangle can be calculated as follows:

$$x^2 + 12^2 = 13^2$$
$$x^2 + 144 = 169$$
$$x^2 + 144 - 144 = 169 - 144$$
$$x^2 = 25$$
$$x = 5$$

Since $\tan\alpha = \frac{\text{opposite leg}}{\text{adjacent leg}}$, $\tan\alpha = \frac{12}{5}$. In addition, $\sin\beta = \frac{8}{17}$, the leg opposite the angle β in a right triangle has a length of 8, while the hypotenuse of the right triangle has a length of 17. For this reason, the other leg of the right triangle can be calculated as follows:

$$x^2 + 8^2 = 17^2$$
$$x^2 + 64 = 289$$
$$x^2 + 64 - 64 = 289 - 64$$
$$x^2 = 225$$
$$x = 15$$

Since $\tan\beta = \frac{\text{opposite leg}}{\text{adjacent leg}}$, $\tan\beta = \frac{8}{15}$.

Since $\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}$,

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$= \frac{\frac{12}{5} + \frac{8}{15}}{1 - \left(\frac{12}{5}\right)\left(\frac{8}{15}\right)}$$
$$= \frac{\frac{36}{15} + \frac{8}{15}}{1 - \frac{96}{75}}$$
$$= \frac{\frac{44}{15}}{\frac{75}{75} - \frac{96}{75}}$$
$$= \frac{\frac{44}{15}}{-\frac{21}{75}}$$
$$= \frac{44}{15} \times \left(-\frac{75}{21}\right)$$
$$= -\frac{3300}{315}$$
$$= -\frac{220}{21}$$

Therefore, the correct answer is **a)**.

4. Since $\sin\theta = \frac{3}{8}$, the leg opposite the angle θ in a right triangle has a length of 3, while the hypotenuse of the right triangle has a length of 8. For this reason, the other leg of the right triangle can be calculated as follows:

$$x^2 + 3^2 = 8^2$$
$$x^2 + 9 = 64$$
$$x^2 + 9 - 9 = 64 - 9$$
$$x^2 = 55$$
$$x = \sqrt{55}$$

Since $\tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}}$,

$$\tan\theta = -\frac{3}{\sqrt{55}} = -\frac{3\sqrt{55}}{55}.$$

(The reason the sign is negative is because angle θ is in the second quadrant.)

Since $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$,

$$\tan 2\theta = \frac{(2)\left(-\frac{3\sqrt{55}}{55}\right)}{1 - \left(-\frac{3\sqrt{55}}{55}\right)^2}$$
$$= \frac{-\frac{6\sqrt{55}}{55}}{1 - \frac{495}{3025}}$$
$$= \frac{-\frac{6\sqrt{55}}{55}}{\frac{3025}{3025} - \frac{495}{3025}}$$

$$= \frac{-\frac{6\sqrt{55}}{55}}{\frac{2530}{3025}}$$
$$= \frac{6\sqrt{55}}{55} \times \frac{3025}{2530}$$
$$= -\frac{18\,150\sqrt{55}}{13\,9150}$$
$$= -\frac{3\sqrt{55}}{23}$$

Therefore, the correct answer is **a)**.

5. Since $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$,

$$\cos\frac{\pi}{8} = \pm\sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}$$
$$= \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$$
$$= \pm\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{2}}$$
$$= \pm\sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}}$$
$$= \pm\sqrt{\frac{2+\sqrt{2}}{4}}$$
$$= \pm\frac{\sqrt{2+\sqrt{2}}}{2}$$

Since the angle $\frac{\pi}{8}$ is in the first quadrant, the sign of $\cos\frac{\pi}{8}$ is positive. Therefore, the correct answer is **d)**.

6. The expression $\frac{2-\sec^2\left(\frac{1}{2}x\right)}{\sec^2\left(\frac{1}{2}x\right)}$ can be simplified as follows:

$$\frac{2-\sec^2\left(\frac{1}{2}x\right)}{\sec^2\left(\frac{1}{2}x\right)} = \frac{2}{\sec^2\left(\frac{1}{2}x\right)} - \frac{\sec^2\left(\frac{1}{2}x\right)}{\sec^2\left(\frac{1}{2}x\right)}$$
$$= 2\cos^2\left(\frac{1}{2}x\right) - 1$$

Since $\cos 2\theta = 2\cos^2\theta - 1$,

$$2\cos^2\left(\frac{1}{2}x\right) - 1 = \cos 2\left(\frac{1}{2}x\right) = \cos x.$$

Therefore, the correct answer is **c)**.

7. The identity $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$ can be proven as follows:

$$\frac{2\tan x}{1+\tan^2 x} = \frac{2\frac{\sin x}{\cos}}{\sec^2 x}$$
$$= 2\frac{\sin x}{\cos x} \times \cos^2 x$$
$$= 2\sin x\cos x$$
$$= \sin 2x.$$

The identities $1+\tan^2 x = \sec^2 x$, $\sin 2x = 2\sin x\cos x$, and $\tan x = \frac{\sin x}{\cos x}$ were all used in the proof.
Therefore, the correct answer is **d)**.

8. The equation $5 + 7\sin\theta = 0$ can be solved as follows:

$$5 + 7\sin\theta = 0$$
$$5 + 7\sin\theta - 5 = 0 - 5$$
$$7\sin\theta = -5$$
$$\frac{7\sin\theta}{7} = -\frac{5}{7}$$
$$\sin\theta = -\frac{5}{7}$$
$$\sin^{-1}(\sin\theta) = \sin^{-1}\left(-\frac{5}{7}\right)$$
$$\theta = \sin^{-1}\left(-\frac{5}{7}\right)$$

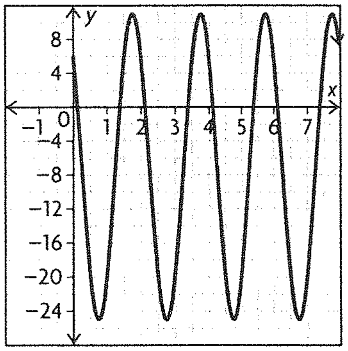
$\theta = -0.80$ or -2.35
Therefore, the correct answer is **b)**.

9. The blade tip is at least 30 m above the ground when $18\cos\left(\pi t + \frac{\pi}{4}\right) + 23 \geq 30$. This inequality can be simplified as follows:

$$18\cos\left(\pi t + \frac{\pi}{4}\right) + 23 \geq 30$$
$$18\cos\left(\pi t + \frac{\pi}{4}\right) + 23 - 30 \geq 30 - 30$$
$$18\cos\left(\pi t + \frac{\pi}{4}\right) - 7 \geq 0$$

The inequality $18\cos\left(\pi t + \frac{\pi}{4}\right) - 7 \geq 0$ can be solved by graphing the function $h(t) = 18\cos\left(\pi t + \frac{\pi}{4}\right) - 7$ and determining where the graph is at or above the x-axis.

The graph is as follows:



The graph is at or above the x-axis in the intervals $1.37 \leq x \leq 2.12$, $3.37 \leq x \leq 4.12$ and $5.37 \leq x \leq 6.12$. Therefore, the correct answer is **c)**.

10. The equation $(2 \sin x + 1)(\cos x - 1) = 0$ is true when either $2 \sin x + 1 = 0$ or $\cos x - 1 = 0$ (or both). If $2 \sin x + 1 = 0$, x can be solved for as follows:

$$2 \sin x + 1 = 0$$
$$2 \sin x + 1 - 1 = 0 - 1$$
$$2 \sin x = -1$$
$$\frac{2 \sin x}{2} = -\frac{1}{2}$$
$$\sin x = -\frac{1}{2}$$

The solutions to the equation $\sin x = -\frac{1}{2}$ occur at $x = 210°$ or $330°$.

If $\cos x - 1 = 0$, x can be solved for as follows:

$$\cos x - 1 = 0$$
$$\cos x - 1 + 1 = 0 + 1$$
$$\cos x = 1$$

The solutions to the equation $\cos x = 1$ occur at $x = 0°$ or $360°$. Therefore, the correct answer is **d)**.

11. Since the solutions to the equation are $\theta = 0$, $\frac{\pi}{3}, \frac{5\pi}{3}$, or 2π, either $\cos \theta = 1$ or $\cos \theta = \frac{1}{2}$. (This is because $\cos 0 = 1$, $\cos \frac{\pi}{3} = \frac{1}{2}$, $\cos \frac{5\pi}{3} = \frac{1}{2}$, and $\cos 2\pi = 1$.) For this reason, either $\cos \theta - 1 = 0$ or $\cos \theta - \frac{1}{2} = 0$ (or both). If the left sides of these two equations are considered factors of a quadratic equation and multiplied together, the result is as follows:

$$(\cos \theta - 1)\left(\cos \theta - \frac{1}{2}\right) = 0$$
$$\cos^2 \theta - \cos \theta - \frac{1}{2} \cos \theta + \frac{1}{2} = 0$$
$$\cos^2 \theta - \frac{3}{2} \cos \theta + \frac{1}{2} = 0$$

If both sides of the equation are multiplied by 2, the result is as follows:

$$(2)\left(\cos^2 \theta - \frac{3}{2} \cos \theta + \frac{1}{2}\right) = (2)(0)$$
$$2 \cos^2 \theta - 3 \cos \theta + 1 = 0$$

Since $2 \cos^2 \theta - 1 = \cos 2\theta$, the equation can be rewritten as follows:

$$2 \cos^2 \theta - 1 + 1 - 3 \cos \theta + 1 = 0$$
$$\cos 2\theta + 1 - 3 \cos \theta + 1 = 0$$
$$\cos 2\theta - 3 \cos \theta + 2 = 0$$

Therefore, the correct answer is **a)**.

12. In the equation $y = \log_7 x$, y is the power to which 7 must be raised in order to produce x. Therefore, the exponential form of $y = \log_7 x$ is $x = 7^y$, and the correct answer is **b)**.

13. In logarithmic functions of the form $f(x) = a \log_{10} [k(x - d)] + c$, if a is negative, $f(x)$ is reflected in the x-axis. Also, If $0 < |k| < 1$, a horizontal stretch of factor $\left|\frac{1}{k}\right|$ occurs, and if $f(x) = \log_{10} x$ a vertical translation of c units up occurs. Therefore, if $b = 1.0117$ is reflected in the x-axis, stretched horizontally by a factor of 3, and translated 2 units up, the resulting function is $f(x) = -\log_{10}\left(\frac{1}{3}x\right) + 2$, and the correct answer is **d)**.

14. Since the value of $\log_7 49$ is the power to which 7 must be raised to produced 49, $\log_7 49 = 2$. Therefore, $7^{\log_7 49} = 7^2 = 49$, so the correct answer is **d)**.

15. The length of the planet's year in days can be calculated as follows:

$$\log_{10} T = 1.5 \log_{10} d - 0.45$$
$$\log_{10} T = 1.5 \log_{10} 11 - 0.45$$
$$\log_{10} T = \log_{10} 11^{1.5} - 0.45$$
$$\log_{10} T - \log_{10} 11^{1.5} = \log_{10} 11^{1.5} - 0.45 - \log_{10} 11^{1.5}$$
$$\log_{10} T - \log_{10} 11^{1.5} = -0.45$$
$$\log_{10} \frac{T}{11^{1.5}} = -0.45$$
$$10^{-0.45} = \frac{T}{11^{1.5}}$$
$$10^{-0.45} \times 11^{1.5} = \frac{T}{11^{1.5}} \times 11^{1.5}$$
$$T = 10^{-0.45} \times 11^{1.5}$$
$$T = 0.3548 \times 36.4829$$
$$T = 12.9$$

Therefore, the correct answer is **c)**.

16. The equation $\log_4 x + 3 = \log_4 1024$ can be solved as follows:

$$\log_4 x + 3 = \log_4 1024$$
$$\log_4 x + 3 - \log_4 x = \log_4 1024 - \log_4 x$$
$$\log_4 1024 - \log_4 x = 3$$
$$\log_4 \frac{1024}{x} = 3$$
$$4^3 = \frac{1024}{x}$$
$$64 = \frac{1024}{x}$$
$$64 \times x = \frac{1024}{x} \times x$$
$$64x = 1024$$
$$\frac{64x}{64} = \frac{1024}{64}$$
$$x = 16$$

Therefore, the correct answer is **a)**.

17. $\log_5 25x = \log_5 25 + \log_5 x$
$= 2 + \log_5 x$

So $g(x)$ is a vertical translation of $f(x)$ 2 units up, and the correct answer is **b)**.

18. The equation $x = \log_3 27\sqrt{3}$ can be rewritten as $3^x = 27\sqrt{3}$. Since $27 = 3^3$ and $\sqrt{3} = 3^{1/2}$, the equation $3^x = 27\sqrt{3}$ can be rewritten as $3^x = 3^3 \times 3^{1/2}$. By adding the exponents on the right side of the equation, the equation becomes $3^x = 3^{3\frac{1}{2}}$. Therefore, x must equal $3\frac{1}{2}$, and the correct answer is **b)**.

19. Since the formula for compound interest is $A = P(1 + i)^n$, the length of time it will take for the investment to be worth more than \$6400 can be calculated as follows:

$$6400 < 1600(1 + 0.01)^n$$
$$\frac{6400}{1600} < \frac{1600(1 + 0.01)^n}{1600}$$
$$4 < (1 + 0.01)^n$$
$$\log_{10} 4 < \log_{10} ((1 + 0.01)^n)$$
$$\log_{10} 4 < n \log_{10} (1 + 0.01)$$
$$n > \frac{\log_{10} 4}{\log_{10} (1 + 0.01)}$$
$$n > \frac{\log_{10} 4}{\log_{10} 1.01}$$
$$n > \frac{0.6021}{0.0043}$$
$$n > 140$$

Since n represents the number of months it will take for the investment to be worth more than \$6400, and since there are 12 months in a year, the number of years it will take for the investment to be worth more than \$6400 is 11 years and 8 months. Therefore, the correct answer is **c)**.

20. Since the formula for the loudness of sound is $L = 10 \log\left(\frac{I}{I_0}\right)$, the intensity of the sound of a jet taking off with a loudness of 133 dB can be calculated as follows:

$$133 = 10 \log\left(\frac{I}{10^{-12}}\right)$$
$$\frac{133}{10} = \frac{10 \log\left(\frac{I}{10^{-12}}\right)}{10}$$
$$\frac{133}{10} = \log\left(\frac{I}{10^{-12}}\right)$$
$$10^{\frac{133}{10}} = \frac{I}{10^{-12}}$$
$$10^{13.3} = \frac{I}{10^{-12}}$$
$$10^{13.3} \times 10^{-12} = \frac{I}{10^{-12}} \times 10^{-12}$$
$$I = 10^{1.3}$$
$$I = 20.0 \text{ W/m}^2$$

Therefore, the correct answer is **d)**.

21. The equation $\log_a (x - 3) + \log_a (x - 2) = \log_a (5x - 15)$ can be rewritten as follows:

$$\log_a (x - 3) + \log_a (x - 2) = \log_a (5x - 15)$$
$$\log_a (x - 3)(x - 2) = \log_a (5x - 15)$$

For this reason, $(x - 3)(x - 2) = 5x - 15$. This equation can be solved as follows:

$$(x - 3)(x - 2) = 5x - 15$$
$$x^2 - 3x - 2x + 6 = 5x - 15$$
$$x^2 - 5x + 6 = 5x - 15$$
$$x^2 - 5x + 6 - 5x + 15 = 5x - 15 - 5x + 15$$
$$x^2 - 10x + 21 = 0$$
$$(x - 7)(x - 3) = 0$$
$$x = 7 \text{ or } x = 3$$

Since it's impossible to find the log of 0, $x = 3$ is not a valid answer, because if 3 is substituted back into the original equation, both sides of the equation would have a term of $\log_a 0$. Therefore, the correct answer is **b)**.

22. Since Carbon–14 has a half-life of 5730 years, the following equation holds true:

$$0.017 = (3.9)\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

This equation can be solved as follows:

$$0.017 = (3.9)\left(\frac{1}{2}\right)^{\frac{x}{5730}}$$

$$\frac{0.017}{3.9} = \frac{(3.9)\left(\frac{1}{2}\right)^{\frac{x}{5730}}}{3.9}$$

$$0.0044 = \left(\frac{1}{2}\right)^{\frac{x}{5730}}$$

$$\log 0.0044 = \log\left(\frac{1}{2}\right)^{\frac{x}{5730}}$$

$$\log 0.0044 = \frac{x}{5730}\log\frac{1}{2}$$

$$-2.3606 = \left(\frac{x}{5730}\right)(-0.3010)$$

$$-2.3606 = -0.000\,05x$$

$$\frac{-2.3606}{-0.000\,05} = \frac{-0.000\,05x}{-0.000\,05}$$

$$x = 44\,933$$

The closest answer is 45 000 years, so the correct answer is **a)**.

23. In each of the next 5 years, the cost of goods and services will increase by 3.1 percent. In other words, in each of the next 5 years, the cost of goods and services will be 103.1 percent of what it was the previous year. Since 103.1 percent written as a decimal is 1.031, the cost of goods and services now should be multiplied by $(1.031)^t$ to find the cost of goods and services after t years. Since the cost of good and services now is P, the correct answer is **c)**.

24. If the population of a city is currently 150 000 and is increasing at 2.3 percent per year, the population of the city in 6 years can be calculated as follows:

$P = (150\,000)(1.023)^6$

$P = (150\,000)(1.1462)$

$P = 171\,927$

Since the population in 6 years will be 171 927, and since the population is increasing at 2.3 percent per year, the number of people by which the population will increase in the 7th year from now will be $(171\,927)(0.023) = 3954$. The closest answer is 4000, so the correct answer is **c)**.

25. It's apparent from the graph that as x moves away from 0 in the negative direction, y becomes smaller and smaller, while as x moves away from 0 in the positive direction, y becomes larger and larger. For this reason, (a) cannot be the correct answer, since as x moves away from 0 in the negative direction, x^2 becomes larger and larger. Also, (b) cannot be the correct answer, since the domain of $\log x$ is $\{x \in \mathbf{R} \mid x > 0\}$, and (d) cannot be the correct answer, since as x moves away from 0 in the positive direction, 0.5^x becomes smaller and smaller. Therefore, the correct answer is **c)**.

26. The domain of $f - g$ is the intersection of the domain of f and the domain of g. Since the domain of f is $\{x \in \mathbf{R} \mid x > 0\}$, and the domain of g is $\{x \in \mathbf{R} \mid x \neq 3\}$, the domain of $f - g$ is $\{x \in \mathbf{R} \mid x > 0, x \neq 3\}$. Therefore, the correct answer is **b)**.

27. The sum or difference of an odd function and an even function is neither even nor odd, unless one of the functions is identically zero, so neither (b) nor (c) can be the correct answer. The difference of two even functions is even, so (d) cannot be the correct answer. Since the sum of two odd functions is always an odd function, the correct answer is **a)**.

28. If $f(x) = g(x) = \sec x$, then $f(x) \times g(x) = \sec^2 x$. Since the range of both $f(x) = \sec x$ and $g(x) = \sec x$ is $\{y \in \mathbf{R} \mid -1 \geq y \geq 1\}$, the range of $f(x) \times g(x) = \sec^2 x$ must be $\{y \in \mathbf{R} \mid y \geq 1\}$. This is because a negative number squared is always positive, and $(-1)^2 = 1$. Therefore, the correct answer is **a)**.

29. If $f(x) = ax^2 + 3$ and $g(x) = bx - 1$, then $(f \times g)(x)$ can be calculated as follows:

$(f \times g)(x) = (ax^2 + 3)(bx - 1)$

$(f \times g)(x) = abx^3 - ax^2 + 3bx - 3$

Since $(f \times g)(x)$ passes through the point $(-1, -3)$, the function can be rewritten as follows:

$$-3 = ab(-1)^3 - a(-1)^2 + 3b(-1) - 3$$

$$-3 = -ab - a - 3b - 3$$

$$-3 + 3 = -ab - a - 3b - 3 + 3$$

$$0 = -ab - a - 3b$$

Also, since $(f \times g)(x)$ passes through the point $(1, 9)$, the function can be rewritten as follows:

$$9 = ab(1)^3 - a(1)^2 + 3b(1) - 3$$

$$9 = ab - a + 3b - 3$$

$$9 + 3 = ab - a + 3b - 3 + 3$$

$$12 = ab - a + 3b$$

If the equations $0 = -ab - a - 3b$ and $12 = ab - a + 3b$ are added together, the resulting equation is $12 = -2a$, or $a = -6$.

If -6 is substituted for a into the equation $0 = -ab - a - 3b$, the equation can be rewritten as follows:

$$\begin{aligned} 0 &= -(-6)b - (-6) - 3b \\ 0 &= 6b + 6 - 3b \\ 0 &= 3b + 6 \\ 0 - 6 &= 3b + 6 - 6 \\ 3b &= -6 \\ b &= -2 \end{aligned}$$

Therefore, the correct answer is **d)**.

30. Since $f(x) = \log x$ and $g(x) = |x - 2|$, $(f \div g)(x) = \dfrac{\log x}{|x-2|}$. Since the denominator can never equal 0, x can never equal 2. Also, since it's impossible to find the log of a number less than or equal to 0, x must be greater than 0. Therefore, the domain of $(f \div g)(x)$ is $\{x \in \mathbf{R} \mid x > 0, x \neq 2\}$, and the correct answer is **d)**.

31. The domain of $f(x) = \sqrt{3 - x}$ is $\{x \in \mathbf{R} \mid x \leq 3\}$. For this reason, the range of $g(x) = 3x^2$ that is permissible is $\{y \in \mathbf{R} \mid y \leq 3\}$, and the domain of $f \circ g$ can be calculated as follows:

$$3x^2 \leq 3$$
$$\frac{3x^2}{3} \leq \frac{3}{3}$$
$$x^2 \leq 1$$
$$-1 \leq x \leq 1$$

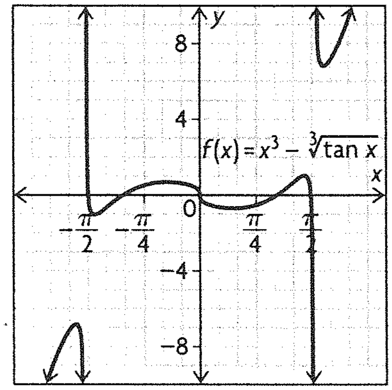
Therefore, the domain of $f \circ g$ is $\{x \in \mathbf{R} \mid -1 \leq x \leq 1\}$, and the correct answer is **c)**.

32. It's clear from the graph that two points on the line are $(0, 3)$ and $(4, -5)$. Since it is already known from the point $(0, 3)$ that the y-intercept of the line is 3, all that is needed to determine the equation of the line is its slope. The slope of the line can be calculated as follows:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{-5 - 3}{4 - 0}$$
$$m = \frac{-8}{4}$$
$$m = -2$$

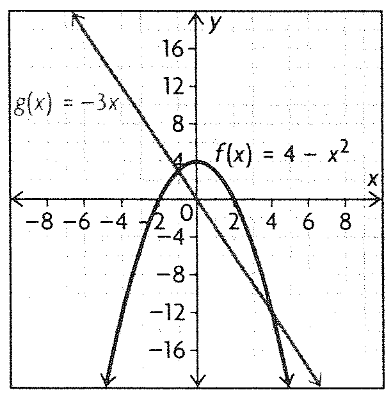
Since the y-intercept of the line is 3 and the slope of the line is -2, the equation of the line is $y = -2x + 3$. Because $(h \circ f)(x) = 3 - 2x = -2x + 3$, the correct answer is **d)**.

33. To solve the equation $x^3 = \sqrt[3]{\tan x}$, first subtract $\sqrt[3]{\tan x}$ from both sides of the equation to produce the equation $x^3 - \sqrt[3]{\tan x} = 0$. Then graph the function $f(x) = x^3 - \sqrt[3]{\tan x}$ and determine the x-coordinates of the points where the graph crosses the x-axis—these are the solutions to the equation. The graph of $f(x) = x^3 - \sqrt[3]{\tan x}$ is as follows:



The graph crosses the x-axis at about $x = -1.55, -1.07, 0, 1.07$, and 1.55. Therefore, the correct answer is **d)**.

34. The functions $f(x) = 4 - x^2$ and $g(x) = -3x$ graphed on the same coordinate grid are as follows:



It's apparent from the graph that $f(x) < g(x)$ when $x < -1$ and when $x > 4$. Therefore, the correct answer is **b)**.

35. Since the horizontal distance that a football can be thrown can be modelled by the function $d = \frac{v^2}{9.8}\sin 2\theta + 1.8$, the angle at which the football was thrown can be calculated as follows:

$$d = \frac{v^2}{9.8}\sin 2\theta + 1.8$$
$$35 = \frac{20^2}{9.8}\sin 2\theta + 1.8$$
$$35 = \frac{400}{9.8}\sin 2\theta + 1.8$$

$$35 - 1.8 = \frac{400}{9.8}\sin 2\theta + 1.8 - 1.8$$
$$33.2 = \frac{400}{9.8}\sin 2\theta$$
$$33.2 \times \frac{9.8}{400} = \frac{400}{9.8}\sin 2\theta \times \frac{9.8}{400}$$
$$\sin 2\theta = 0.8134$$
$$\sin^{-1}(\sin 2\theta) = \sin^{-1}(0.8134)$$
$$2\theta = 54.4295° \text{ or } 125.5705°$$
$$\frac{2\theta}{2} = \frac{54.4295°}{2} \text{ or } \frac{125.5705°}{2}$$
$$\theta = 27° \text{ or } 63°$$

Therefore, the football could have been thrown at either 27° or 63° relative to the horizontal.

36. a) Answers may vary. For example, since population growth is usually exponential, suitable models for the population of Niagara and Waterloo could be exponential functions in the form $P(x) = ab^x$. If the year 1996 is considered $t = 0$, the model for Niagara can be developed as follows:

$P(x) = ab^x$
$414.8 = ab^0$
$414.8 = a(1)$
$a = 414.8$

Since $a = 414.8$, the model can now be developed as follows:

$P(x) = (414.8)(b^x)$
$476.8 = (414.8)(b^{32})$
$\frac{476.8}{414.8} = \frac{(414.8)(b^{32})}{414.8}$
$b^{32} = 1.1495$
$b = 1.0044$

Since $b = 1.0044$, the model for Niagara is $P(x) = (414.8)(1.0044^x)$.

The model for Waterloo can be developed as follows:

$P(x) = ab^x$
$418.3 = ab^0$
$418.3 = a(1)$
$a = 418.3$

Since $a = 418.3$, the model can now be developed as follows:

$P(x) = (418.3)(b^x)$
$606.1 = (418.3)(b^{32})$
$\frac{606.1}{418.3} = \frac{(418.3)(b^{32})}{418.3}$
$b^{32} = 1.4490$
$b = 1.0117$

Since $b = 1.0117$, the model for Waterloo is $P(x) = (418.3)(1.0117^x)$.

b) Answers may vary. For example, to estimate the doubling time for Niagara, the model $P(x) = (414.8)(1.0044^x)$ could be used as follows:

$$P(x) = (414.8)(1.0044^x)$$
$$829.6 = (414.8)(1.0044^x)$$
$$\frac{829.6}{414.8} = \frac{(414.8)(1.0044^x)}{414.8}$$
$$1.0044^x = 2$$
$$\log 1.0044^x = \log 2$$
$$(x)(\log 1.0044) = \log 2$$
$$\frac{(x)(\log 1.0044)}{\log 1.0044} = \frac{\log 2}{\log 1.0044}$$
$$x = \frac{0.3010}{0.0019}$$
$$x = 159 \text{ years}$$

To estimate the doubling time for Waterloo, the model $P(x) = (418.3)(1.0117^x)$ can be used as follows:

$$P(x) = (418.3)(1.0117^x)$$
$$836.6 = (418.3)(1.0117^x)$$
$$\frac{836.6}{418.3} = \frac{(418.3)(1.0117^x)}{418.3}$$
$$1.0117^x = 2$$
$$\log 1.0117^x = \log 2$$
$$(x)(\log 1.0117) = \log 2$$
$$\frac{(x)(\log 1.0117)}{\log 1.0117} = \frac{\log 2}{\log 1.0117}$$
$$x = \frac{0.3010}{0.0050}$$
$$x = 60 \text{ years}$$

c) Answers may vary. For example, to calculate the rate at which Niagara's population will be growing in 2025, first it's necessary to find the projected population in 2024, and then it's necessary to find the projected population in 2025. The projected population in 2024 can be found as follows:

$P(x) = (414.8)(1.0044^{28})$
$P(x) = (414.8)(1.1308)$
$P(x) = 469.1$

The projected population in 2025 can be found as follows:

$P(x) = (414.8)(1.0044^{29})$
$P(x) = (414.8)(1.1358)$
$P(x) = 471.1$

Therefore, the rate at which Niagara's population will be growing in 2025 is $471.1 - 469.1 = 2$ thousand people per year. To calculate the rate

at which Waterloo's population will be growing in 2025, first it's necessary to find the projected population in 2024, and then it's necessary to find the projected population in 2025. The projected population in 2024 can be found as follows:

$P(x) = (418.3)(1.0117^{28})$
$P(x) = (418.3)(1.3850)$
$P(x) = 579.3$

The projected population in 2025 can be found as follows:

$P(x) = (418.3)(1.0117^{29})$
$P(x) = (418.3)(1.4012)$
$P(x) = 586.1$

Therefore, the rate at which Waterloo's population will be growing in 2025 is $586.1 - 579.3 = 6.8$ thousand people per year. Since Niagara's population will be growing at 2 thousand people per year and Waterloo's population will be growing at 6.8 thousand people per year, Waterloo's population will be growing faster.

37. Since the mass of the rocket just before launch is 30 000 kg, and since its mass is decreasing at 100 kg/s, $m(t) = 30\,000 - 100t$. Since $T - 10m = ma$, a can be isolated as follows:

$$T - 10m = ma$$
$$\frac{T - 10m}{m} = \frac{ma}{m}$$
$$a = \frac{T - 10m}{m}$$
$$a = \frac{T}{m} - 10$$

Since $m(t) = 30\,000 - 100t$, $a(t)$ can be determined as follows:

$$a = \frac{T}{m} - 10$$
$$a(t) = \frac{T}{30\,000 - 100t} - 10$$

Since $m = 30\,000(2.72)^{-v-gt}$, v can be isolated as follows:

$$m = 30\,000(2.72)^{-v-gt}$$
$$\frac{m}{30\,000} = \frac{30\,000(2.72)^{-v-gt}}{30\,000}$$
$$\frac{m}{30\,000} = (2.72)^{-v-gt}$$
$$\log\frac{m}{30\,000} = \log((2.72)^{-v-gt})$$
$$\log\frac{m}{30\,000} = (-v - gt)(\log 2.72)$$
$$\frac{\log\frac{m}{30\,000}}{\log 2.72} = \frac{(-v - gt)(\log 2.72)}{\log 2.72}$$
$$-v - gt = \frac{\log\frac{m}{30\,000}}{\log 2.72}$$
$$-v - gt + gt = \frac{\log\frac{m}{30\,000}}{\log 2.72} + gt$$
$$-v = \frac{\log\frac{m}{30\,000}}{\log 2.72} + gt$$
$$-v \times -1 = \left(\frac{\log\frac{m}{30\,000}}{\log 2.72} + gt\right) \times -1$$
$$v = -\frac{\log\frac{m}{30\,000}}{\log 2.72} - gt$$

Since $m(t) = 30\,000 - 100t$, $v(t)$ can be determined as follows:

$$v = -\frac{\log\frac{m}{30\,000}}{\log 2.72} - gt$$
$$v(t) = -\frac{\log\frac{30\,000 - 100t}{30\,000}}{\log 2.72} - gt$$
$$v(t) = -\frac{\log\left(1 - \frac{t}{300}\right)}{\log 2.72} - gt$$

In order for the rocket to get off the ground, $a(0)$ must be greater than 0. Therefore, the constraints on the value of T can be determined as follows:

$$a(t) = \frac{T}{30\,000 - 100t} - 10$$
$$0 < \frac{T}{30\,000 - 100(0)} - 10$$
$$0 < \frac{T}{30\,000} - 10$$
$$0 + 10 < \frac{T}{30\,000} - 10 + 10$$
$$10 < \frac{T}{30\,000}$$
$$10 \times 30\,000 < \frac{T}{30\,000} \times 30\,000$$
$$T > 300\,000 \text{ N}$$